Trajectory Optimization for High-Speed Trains via a Mixed Integer Linear Programming Approach

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***Abstract*— This paper proposes a trajectory optimization approach for high-speed trains to reduce traction energy consumption and increase riding comfort. Besides, the proposed approach can also achieve energy-saving effects by optimizing the operation time between stations. First, an optimization model is developed by defining the objective function as a trade-off function of the traction energy consumption and riding comfort. In addition to constraints in the classic optimal train control model, three new factors−the discrete throttle settings, neutral zones, and sectionalized tunnel resistance−are considered. Then, the model is discretized and turned into a multi-step decision optimization problem. All the nonlinear constraints are approximated using piecewise affine (PWA) functions, and the trajectory optimization problem is turned into a mixed integer linear programming (MILP) problem which can be solved by existing solvers CPLEX and YALMIP. Finally, some case studies with real-world data sets are conducted to present the effectiveness of the proposed approach. The simulation results are compared with the practical running data of trains, which shows that the proposed model and the optimization approach save energy and improve the riding comfort.**

***Index Terms*— Energy efficiency, MILP, train control, riding comfort, high-speed railway.**

I. Introduction

IN THE past decades, high-speed rail is developing fast all over the world. Many countries including China, Japan, Germany, France, and Spain have built a high-speed network which reduces the trip time between big cities and promotes the economy of these areas. In China, the biggest high-speed network in the world consists of four vertical and four horizontal lines and the total mileage has been over 35 thousand kilometers by the end of 2019. Trains reach speeds of up to $350km/h$ and the trip time from Beijing to Shanghai has decreased from 12 hours to only 4.5. The annual number of passengers for high-speed rail is about 2 billions in 2019, accounting for 64% of the whole railway system. This ratio is still increasing by 10% per year. In 2035, a bigger high-speed network including eight vertical and eight horizontal lines will be constructed in China.

With the increasing scale of high-speed railway, the energy efficiency of railways is paid more attention due to the rising energy prices and environmental concerns [5], [23]. The traction energy consumption occupies a large portion of the total energy consumption, and the energy increases by 90% when the maximum train speed rises from $300km/h$ to $350km/h$. This suggests that reducing traction energy consumption by using energy-efficient driving strategies can greatly reduce the energy for high-speed railways. Railway systems aim to provide service to passengers, so the riding comfort is of great importance for a long journey. More switching and the associated jerking during the train operation reduce riding comfort for the passengers. The automatic train operation (ATO) system that automatically controls the traction and braking of trains plays an important role in ensuring precise stopping, operation punctuality, energy efficiency, and riding comfort [25]. Specifically, the ATO system calculates a speed trajectory, and the train runs by following the trajectory with a chosen control principle [20]. Therefore, the speed trajectory determines the traction energy consumption and the riding comfort in some sense. Hence, an optimization algorithm that calculates an efficient planned train trajectory according to the information collected by the train control system can improve both energy efficiency and riding comfort [27].

Classic train operation models consider features such as speed limits and gradient. These model neglect relevant characteristic of high-speed railways such as the alternating current (AC), neutral zones, tunnel resistance and discrete throttle settings. First, the high-speed railway is powered by AC, which implies the existence of neutral zones. The neutral zones isolate the different phases of electricity supplied by different substations through two phase-splitting switches, preventing the short circuiting of different phases and the fusing of catenaries. When trains run through the neutral zone, there is no electricity. It means that traction and electric braking cannot be applied in these areas [35]. Second, the tunnel resistance

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when trains run at a high speed will be very large and strongly nonlinear. Travelling through a tunnel can be divided into three stages, i.e., entering, travelling through and exiting [28]. The aerodynamics in each stage are different, but this has not been considered in the classic model. The last factor we will add to the model is the discrete throttle settings. An operator can only use these fixed throttles and traction motor to output the corresponding power. Adding the factors of neutral zones, stages of tunnel transit, and discrete throttle settings will bring some challenges for the train control problem when the energy consumption, riding comfort, punctuality are all considered.

This paper proposes a trajectory optimization approach for high-speed trains based on the MILP methodology with considering the characteristics of the high-speed railways. The main contributions of this paper are listed as follows.

- Considering the real line conditions of high-speed trains, such as tunnels and neutral zones, a new train nonlinear optimization control model based on discrete control is proposed. Next, the above nonlinear optimization control problem is transformed into a mixed integer linear programming (MILP) problem.
- For discretely controlled trains, the influence of riding comfort on the energy-efficient driving strategy of trains is analyzed, especially on the quasi-coasting during the trip.
- Combined with actual high-speed train operation data, the proposed method is simulated and verified. We analyze the effect of linearization errors introduced in our conversion to the MILP model.

The rest of this paper is organized as follows. In Section II, the literature on the train control problem is reviewed. In Section III, the optimal train control model for high-speed trains is developed. The optimal train control problem is discretized and transformed into a MILP problem in Section IV. Then, the solution is obtained by the existing CPLEX solver. In Section V, six case studies are conducted based on the real-world operational data to illustrate the effectiveness of the proposed approach. Conclusions are given at the end.

II. Literature Review

The study of trajectory optimization for trains has began since the 1960s. Ichikawa [1] solved the train optimal control problem using the Pontryagin principle firstly, in which it was assumed that the train runs on a flat track with constant gradient and traction efficiency. The optimal driving strategy is proved to consist of maximum acceleration, cruising, coasting and maximum braking, which builds the foundation of the optimal train control theory. Later on, a lot of researchers studied the optimal planning of the train trajectory and gradually took the variable slopes, arbitrary speed limits and curve resistance into consideration. Howlett [2] considers the optimal driving strategy in a train with continuous and discrete control problem and pays particular attention to a unified development of the two cases. For continuous control problem, Pontryagin principle is used to find necessary conditions on an optimal strategy and show that these conditions yield key equations that determine the optimal switching points. In addition, this paper also study the optimal driving strategy of the train when there is a steep uphill or a steep downhill in the line, and gave a formula to calculate switching points into and out of maximum acceleration when traversing a steep uphill section or into and out of coast when traversing a steep downhill section. For discrete control problem, Karush-Kuhn-Tucker (KKT) conditions are used to find key equations that determine the optimal switching times. The main contribution of this paper is to prove that the optimal strategies type of discrete control can be used to as closely as that obtained using continuous control. Khmelnitsky [3] constructed a numerical algorithm which exploits some properties of the optimal solution obtained from the Pontryagin principle analysis to minimizes energy consumption in moving the train along a given route for a given time. The uniqueness of this paper is that a new model of the train is constructed, in which kinetic energy and time are used as state variables of the train, and position is used as independent variables. The highlights of this paper is to deal with three difficulties simultaneously, (a) grade profile is an arbitrary variable function of the way, (b) speed restriction is also an arbitrary variable function, (c) tractive and brake forces depend on velocity. Liu and Golovitcher [4] analyze the optimal driving regimes of the train including maximum traction, cruising, coasting, and maximum braking and developed a computer program for energy-efficient train control. The proposed program can not only calculate the optimal driving strategy of the train between two stations, but also developing energy-efficient operating schedules by optimizing distributions of running time for an entire route. An important conclusion drawn in this paper is that for a given line condition and a given running time, there is only one corresponding cruising speed under an optimal driving strategy. Besides, when regenerative braking is considered, Howlett *et al.* [6] discussed formulation of the model, determined the characteristic optimal control modes, studied allowable control transitions, established the existence of optimal switching points and considered optimal strategies with speed limits. Next, Howlett [7] established important integral forms of the necessary conditions for optimal switching, found general bounds on the positions of the optimal switching points, justified the local energy minimization principle and showed how these ideas are used to calculate optimal switching points. The above two literatures prove the fact that an optimal strategy always exists and use a perturbation analysis to show the strategy is unique.

Pontryagin principle is only used to the analysis of continuous optimal control problems. For a train with discrete control input, i.e. the traction/braking force can noly take some discrete values, Howlett [8]–[12] used the KKT conditions to determine the optimal switching times or switching points, for a given fixed discrete valued control sequence, with the continuously varying gradient, non-zero track gradient, and speed limits are considered.

In addition, some numerical methods are used to study the optimal driving strategy of trains. Ko *et al.* [13] investigated an algorithm for optimizing the train trajectory with Bellman's Dynamic programming (DP). They found that many methods often meet some difficulties when more realistic conditions (such as local constraints of state variables, nonlinear running resistance, variable grade profile) are considered. DP method can deal with this complicated conditions. Wang [14] applied the pseudospectral method and MILP approach to solve the optimal trajectory planning problem for trains. In the pseudospectral method, the optimal trajectory planning problem is formulated as a multiple-phase optimal control problem based on piecewise line resistance and speed limits. And in the MILP approach, the nonlinear train operation model is formulated as a mixed logical dynamical model by using PWA functions. Furthermore, the optimal control problem is recast as an MILP problem, which can be solved efficiently by existing solvers. The result shows that the pseudospectral method presents a better control performance, but it has longer computation time. The MILP approach yields the best overall performance.

Besides, some searching method for optimizing the train trajectory was proposed. Chang and Sim [22] used the genetic algorithm to solve the optimization problem, in which the punctuality, riding comfort and energy consumption are taken as the objectives. A coasting control methodology is applied in this paper and the control variable is defined to be the switching points between the acceleration and coasting regimes. Ke *et al.* [24] proposed an ant colony optimization algorithm to optimize the train trajectory and designed a combinatorial optimization method to reduce the calculation time.

In addition to the main methods mentioned above, there are some other methods to solve the optimal control problem of trains. Su [19] reanalyzed the optimal train control model with considering the traction efficiency. Then, a target speed profile design approach is proposed for the ATO system [21]. Su [17], [19], [21] extended the train trajectory optimization problem and proposed an integrated timetable optimization approach, which can systematically reduce the traction energy consumption. Miyatake [29], [30] reformulated an optimal train control model by taking the energy storage into consideration. A sequential quadratic programming (SQP) algorithm was proposed to find the solution of the problem.

High-speed railway has some special characteristics and the corresponding practical model are distinguished from the above proposed model. For example, special line conditions (such as neutral zones and tunnels) will have a great influence on the train operation for high-speed trains. The running resistance when trains are running through the tunnel will be very large and strongly nonlinear. There is no electricity in the neutral zone, which will bring strong constraints to the optimal train control problem. Hence, the neutral zones, tunnel resistance and discrete throttle settings of trains are considered in this paper to improve the optimal train control theory.

III. Model Formulation

For a better understanding of this paper, the main symbols used in this paper are firstly presented in Table I.

TABLE I
SYMBOLS

Symbol	Description	Symbol	Description
E	Traction energy consumption	t	Time index
ρ	Rotating mass factor	u	Traction force
s	Position of the train	L_R	Curve length
w	Additional resistance	B	Braking force
g	Gradient resistance	P_d	Riding comfort
a	Train acceleration	T	Trip time
f_c	Curve resistance	R	Curve radius
f_t	Tunnel resistance	$\overline{V(s)}$	Speed limits
r	Running resistance	v	Train speed
φ	Weighting factor	L_T	Train length
h	Applied throttle	–	–

For a fixed line and vehicle, the train trajectory determines the traction energy consumption of trains on the line with respect to the given time. The traction energy consumption is generally calculated to be

$$E = \int_0^T \frac{u(t) + |u(t)|}{2} v(t)\mathrm{d}t. \tag{1}$$

Another objective for the control of the high-speed train is the riding comfort which can be obtained by the integral of the jerk during the train operation.

$$P_d = \int_0^T |\frac{da(t)}{dt}|\mathrm{dt}. \tag{2}$$

Hence, the objective function for optimizing the trajectory of the high-speed train is concluded as

$$\min Q = \int_0^T [\frac{u(t) + |u(t)|}{2} v(t) + \varphi |\frac{da(t)}{dt}|]\mathrm{d}t. \tag{3}$$

The motion of the high-speed train can be described by the motion of a point mass train, which is written as

$$\rho \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{u(t) - B(t) - r(v) - w(v,s)}{m}, \quad \frac{\mathrm{d}s}{\mathrm{d}t} = v. \tag{4}$$

Taking the practical conditions into consideration, the variables in equation (4) satisfy the following properties:

$$r'(v) > 0, \quad r''(v) > 0, \tag{5}$$

$$B_{max}(t) + r(v) + w(v,s) > 0. \tag{6}$$

Equation (6) presents the fact that the running train can stop at any position on the route.

The applied force of the train should satisfy the constraints from the characteristics of the vehicle traction and braking forces.

$$u_{min}(v) \le u(t) \le u_{max}(v). \tag{7}$$

The braking force also should follow the constraint from the vehicle.

$$0 \le B(t) \le B_{max}(v). \tag{8}$$

Besides, discrete throttle settings are set in the high-speed trains. It is assumed that there are "n" throttles for the high-speed trains and the traction or braking forces are the function of each throttle.

$$u(t) = Z_i[h(t)], \quad \text{if} \quad h(t) = i, \ i = 1, \ldots, n. \tag{9}$$

Trains cannot apply traction forces in the neutral zones, the traction force constraint is shown as follows.

$$u(s) = 0, \quad s_{j_1} \leq s \leq s_{j_2}, \tag{10}$$

where s_{j_1} and s_{j_2} are the initial and final position of the "j"th neutral zone.

The additional resistance consists of the gradient force, curve resistance, and tunnel resistance.

$$w(v, s) = g(s) + f_c(s) + f_t(v). \tag{11}$$

The gradient force is generally calculated with the following equation when the train is considered as a mass point.

$$g(s) = mg \sin\theta(s). \tag{12}$$

The curve resistance in railway system can be described as the function of the radius of the curve and the length of the train.

$$f_c(s) = \begin{cases} \dfrac{k}{R_i(s)}, & L_T \leq L_{R_i}, \\ \dfrac{kL_{R_i}}{R_i(s)L_T}, & L_T > L_{R_i}. \end{cases} \tag{13}$$

The tunnel resistance is a piecewise function which is related to the relative position between the train and the tunnel. Traveling through a tunnel can be divided into three stages, i.e., part of the train enters the tunnel, the whole train run in the tunnel, part of the train exits the tunnel [26], [28]. And the sectionalized tunnel resistance is generally calculated to be [34]

$$f_t(v) = \begin{cases} a_1 + b_1 v + c_1 v^2, & s_{t_{c_1}} < s \leq s_{t_{c_1}} + L_T, \\ a_2 + b_2 v + c_2 v^2, & s_{t_{c_1}} + L_T < s \leq s_{t_{c_2}}, \\ a_3 + b_3 v + c_3 v^2, & s_{t_{c_2}} < s \leq s_{t_{c_2}} + L_T, \end{cases} \tag{14}$$

where $s_{t_{c_1}}$ and $s_{t_{c_2}}$ are the initial and final position of the "t"th tunnel.

During the train operation, the train speed at any position cannot higher than the speed limits which are generally determined by the infrastructure or temporary settings.

$$v(s) \leq \overline{V(s)}. \tag{15}$$

Since the train will depart at one station and stop in the terminate station and the boundary conditions of the high-speed train operation are

$$s(0) = s_0, \quad s(T) = s_T, \quad v(0) = v_0, \quad v(T) = v_T, \tag{16}$$

where, v_0 and v_T are the initial and final speed at the corresponding positions at s_0 and s_T in the route.

IV. Solution

A. Discrete Model

It is shown that the speed and time of the train are state variables, and the position of the train is independent variables, and the traction and braking force are the control variables. The constraints in the train control model is divided into two types, i.e., the constraints related to the position and the time. To transform the train trajectory optimization model into a MILP problem, the train kinetic energy "E" instead of speed "v" is chosen as the state variable, which aims to linearize the nonlinear constraints.

The distance is divided into N intervals and the problem is turned into a multi-step decision optimization model. The number of kinetic energy and time point are $N+1$. $E(k+1)$ and $t(k+1)$ are initial kinetic energy and moment when the train arrives at the position $s(k+1)$ as well as the final kinetic energy and moment for the position $s(k)$. $u(k)$ is the traction force in the segment $[s(k), s(k+1)]$, and it is assumed that the line data, vehicle parameters, and the traction and braking forces are constant in each segment. $\Delta s(k)$ is the length of kth interval. $u(k)\Delta s(k)$ means the traction energy consumption in the segment $[s(k), s(k+1)]$. $\xi(k)$ is defined as the absolute value of the change of adjacent control throttle with respect to the train position, which can be an approximate measure of riding comfort [15].

The objective function is reformulated to be

$$Q = \sum_{k=1}^{N} u(k)\Delta s(k) + \sum_{k=1}^{N-1} \varphi\xi(k), \tag{17}$$

where $\varphi > 0$ is a suitable constant. The $\xi(k)$ is defined as $|h(k+1) - h(k)|$.

Here, the unit mass is considered in this paper. Equation (4) can be rewritten as:

$$\rho(\Delta E(k)) = u(k) - B(k) - r(\sqrt{2E(k)}) - w(s(k), \sqrt{2E(k)}), \tag{18}$$

$$\Delta t(k) = \frac{s(k)}{\sqrt{2E(k)}}, \tag{19}$$

where $\Delta E(k) = E(k+1) - E(k)$, $r(\sqrt{2E(k)})$ is the basic resistance in the interval $[s(k), s(k+1)]$, which includes rolling resistance and air resistance. $w(s(k), \sqrt{2E(k)})$ is the additional resistance in the interval $[s(k), s(k+1)]$. In practice, $r(\sqrt{2E(k)})$ and $w(s(k), \sqrt{2E(k)})$ are usually given by empirical formulas. $\Delta t(k) = t(k+1) - t(k)$.

The constraints on the sectionalized tunnel resistance can be expressed by the following function with kinetic energy as a variable.

$$f_t(\sqrt{2E}) = \begin{cases} a_1' + b_1'\sqrt{2E} + c_1'E, & s_{t_{c_1}} < s \leq s_{t_{c_1}} + L_T \\ a_2' + b_2'\sqrt{2E} + c_2'E, & s_{t_{c_1}} + L_T < s \leq s_{t_{c_2}} \\ a_3' + b_3'\sqrt{2E} + c_3'E, & s_{t_{c_2}} < s \leq s_{t_{c_2}} + L_T. \end{cases} \tag{20}$$

The speed limit constraint is reformulated to be

$$0 < E(s) \leq \overline{E(s)}. \tag{21}$$

An assumption should be noted that the velocity of train is always larger than zero, which is not a harsh term in practice for two reasons stated by Khmelnitsky [3]. First, the initial kinetic energy (E_{start}) and the terminal kinetic energy (E_{end}) are approximated by a small positive number. Second, the train will not stop at the intermediate point of the journey [3].

B. Linearization

In this subsection, all nonlinear constraints of the trajectory optimization model is transformed into linear constraints such that the original problem is reformulated to be a MILP problem.

Using the three transformation properties introduced in Equation (33) (34) (35) of Literature [14] and PWA functions [33], the trajectory optimization model is transformed into a MILP model.

PWA approximation is a general method which can transform the nonlinear function into a linear function [14]. Taking the used function $f_1(E) = \frac{1}{2\sqrt{2E}}$ for example, three affine subfunctions can be used to approximate the original function. The PWA subfunctions are:

$$\hat{f}_1(E(k)) = \begin{cases} \alpha_4 E(k) + \beta_4, & \text{for} \quad E_0 \leq E \leq E_1, \\ \alpha_5 E(k) + \beta_5, & \text{for} \quad E_1 \leq E \leq E_2, \\ \alpha_6 E(k) + \beta_6, & \text{for} \quad E_2 \leq E \leq E_3, \end{cases} \tag{22}$$

where E_0 and E_3 are the minimum and maximum kinetic energy of trains and the parameters $\{\alpha_i, i = 4, 5, 6\}$ can be determined with the classical least squares method. It is also noticed that the approximate error can be reduced when more subfunctions are used as the PWA functions.

With the stated three properties in Literature [14] and the PWA method, the trajectory optimization model is linearized as the following.

The time constraint is obtained by solving the differential equation. On the basis of trapezoidal integration rule [16], equation (19) can be rewritten as the following:

$$t(k+1) = t(k) + (\frac{1}{2\sqrt{2E(k)}} + \frac{1}{2\sqrt{2E(k+1)}})s(k), \tag{23}$$

for $k = 1 \ldots N$, $f_1(E(k)) = \frac{1}{2\sqrt{2E(k)}}$ can be rewritten as the following equation with logical variable δ, $\delta \in \{0,1\}$.

$$\begin{aligned} \hat{f}_1(E(k)) = & \begin{bmatrix} -\alpha_6 & \alpha_5 - \alpha_6 & \alpha_4 - \alpha_5 + \alpha_6 \end{bmatrix} \\ & \times \begin{bmatrix} z_4(k) & z_5(k) & z_6(k) \end{bmatrix}^T \\ & + \begin{bmatrix} -\beta_6 & \beta_5 - \beta_6 & \beta_4 - \beta_5 + \beta_6 \end{bmatrix} \\ & \times \begin{bmatrix} \delta_4(k) & \delta_5(k) & \delta_6(k) \end{bmatrix}^T \\ & + \alpha_6 E(k) + \beta_6, \end{aligned} \tag{24}$$

for k = 1,…,N. $\delta_4(k)$, $\delta_5(k)$, $\delta_6(k)$ are logical variables. $\delta_6(k) = \delta_4(k)\delta_5(k)$, $z_4(k) = \delta_4(k)E(k)$, $z_5(k) = \delta_5(k)E(k)$, $z_6(k) = \delta_6(k)E(k)$. To satisfy the running time constraint, an additional constraint for the time is:

$$t(1) = 0, \quad t(N+1) \leq T. \tag{25}$$

Considering equation (7), u_{min} is considered as a constant in this paper. In the same way, the maximum traction force $u_{max}(v)$ is normally described as a nonlinear formula, which can also be approximated by PWA functions [14].

$$\begin{aligned} u_{max}(E(k)) = & [-\alpha_3 \quad \alpha_2 - \alpha_3 \quad \alpha_1 - \alpha_2 + \alpha_3] \\ & \times [z_1(k) \quad z_2(k) \quad z_3(k)]^T \\ & + [-\beta_3 \quad \beta_2 - \beta_3 \quad \beta_1 - \beta_2 + \beta_3] \\ & \times [\delta_1(k) \quad \delta_2(k) \quad \delta_3(k)]^T \\ & + \alpha_3 E(k) + \beta_3, \end{aligned} \tag{26}$$

for $k = 1, \ldots, N+1$, where $\delta_1(k)$, $\delta_2(k)$, $\delta_3(k)$ are the logical variable, $z_1(k) = \delta_1(k)E(k)$, $z_2(k) = \delta_2(k)E(k)$, $z_3(k) = \delta_3(k)E(k)$, $\delta_3(k) = \delta_1(k)\delta_2(k)$. $u(k)$ should also subject to:

$$u(k) \leq min\{u_{max}(E(k)), u_{max}(E(k+1)\}, \tag{27}$$

for $k = 1, \ldots, N-1$.

For the neutral zones, the train must have enough energy which can support the train running until it leaves the neutral zone. Meanwhile, to improve the riding comfort, the traction may not be too large before it runs into the neutral zone [18]. Ideally, the throttles would be stepped down to zero, such as 5, 4, 3, 2, 1, 0. The constraints for the neutral zone can be rewritten as following functions.

$$u(s) = 0, \quad s_{n_1} \leq s \leq s_{n_2}, \tag{28}$$

where s_{n_1} and s_{n_2} are the start and end positions of the "n"th neutral zones, respectively.

The constraints on riding comfort can be expressed by the following inequality.

$$|h(k+1) - h(k)| \leq 1, \quad k = 1, \ldots, N-1, \tag{29}$$

which can be turned into two inequalities as:

$$\begin{cases} h(k+1) - h(k) \leq 1, \\ h(k+1) - h(k) \geq -1. \end{cases} \tag{30}$$

It is noticed that the objective function can be reformulated only with the calculation of the energy of the train if the constraints of equations (30) are considered.

In reality, the air resistance which is related to the quadratic term of speed is much larger than the linear term, especially for the high-speed trains. Hence, to simplify the model, the linear resistance with respect to the kinetic energy is used in this paper as the following format [31]:

$$r(\sqrt{2E(k)}) = r_1 + r_2 E(k), \tag{31}$$

for $k = 1, \ldots, N$, where r_1 and r_2 are determined by the characteristics of the train.

In the same way, the additional resistance can be approximated by the the following equation.

$$w(s(k), \sqrt{2E(k)}) = w_1 + w_2 E(k), \tag{32}$$

for $k = 1, \ldots, N$.

Then, equation (18) can be written as:

$$\rho(\Delta E(k)) = u'(k) - (r_1 + r_2 E(k)) - (w_1 + w_2 E(k)), \tag{33}$$

where $u'(k)$ is traction or braking force of the train.

It has been shown that the following function can be obtained after solving differential equation.

$$\begin{aligned} E(k+1) = & e^{\mu_k \Delta s_k} E(k) + (e^{\mu_k \Delta s_k} - 1)\frac{\theta}{\mu_k} u(k) \\ & + (e^{\mu_k \Delta s_k} - 1)\frac{\sigma_k}{\mu_k}, \end{aligned} \tag{34}$$

for k = 1,…,N, where

$$\theta = \frac{1}{\rho}, \quad \mu_k = -\frac{2\,(r_2 + w_2)}{\rho}, \quad \sigma_k = -\frac{r_1 + w_1}{\rho}. \tag{35}$$

TABLE II
PARAMETERS OF THE VEHICLE

Definition	Symbol	Value
Train mass[kg]	m	$4.70*10^5$
Resistance coefficient	c_1	0.034
Resistance coefficient	c_2	$2.642 * 10^{-5}$
Rolling factor	ρ	1.06
Maximum velocity[km/h]	V_{max}	350
Minimum kinetic energy [J]	E_{min}	0.1
Maximum traction force[KN]	u_{max}	258.5
Maximum braking force[KN]	u_{min}	-248.5
Train length[m]	L_T	200

TABLE III
THE COEFFICIENTS OF THE PWA APPROXIMATION

n	α_n	β_n	E
1	$-0.1951 * 10^{-3}$	0.9120	0-161
2	$-0.2149 * 10^{-3}$	0.8834	161-890
3	$-0.2408 * 10^{-4}$	0.4364	890-4800
4	$-0.3509 * 10^{-3}$	0.0799	0-756
5	$-0.1613 * 10^{-4}$	0.0256	756-2400
6	$-0.3220 * 10^{-5}$	0.0141	2400-5000

Meanwhile, the boundary conditions for $E(k)$ is that:

$$E(1) = E_{start}, \quad E(N+1) = E_{end}. \tag{36}$$

Above all, the nonlinear constraints have been turn into linear constraints. The objective function is the equation (17), constraints are with respect to equations (22-36). The trajectory planning optimization problem has been turned into a MILP problem, which can be solved by several solvers, such as Cplex, Xpress-MP, GLPK etc.

V. CASE STUDY

To illustrate the effectiveness of the proposed approach, some numerical cases are conducted in this section with the real-world data of Beijing-Shanghai high-speed line in China. The simulated interval is from Beijing to Cangzhou station. The total distance of the interval is 209.97 km, which covers three segments and four stations i.e., Beijing South station, Langfang station, Tianjin South station and Cangzhou West station. The parameters of the vehicle are listed in Table II. And the parameters of PWA approximation used in this paper are shown in Table III.

First, the optimization results of the algorithm proposed in this paper are compared with the actual operation data in Case 1. Second, the impact of the neutral zones on driving strategy is analyzed in Case 2. Third, the influence of riding comfort and sensitiveness of the parameters of riding comfort are analyzed in Case 3. Then in Case 4, the effect of the tunnel on driving strategy is studied. In Case 5, the difference between multiple segments optimization and single segment optimization are compared. Finally, the linearization error in the model is analyzed in the last case.

A. Case 1

In order to further verify the effectiveness of the algorithm proposed in this paper, the trajectory which is optimized by MILP algorithm and the actual running trajectory are compared. The actual operating data comes from the $G333$ train on October 23, 2020 (the driver is Liu Bo). When the given time is $1226s$ and weight factor φ is 200, the model cost using the driver implemented speed profile and using recommended speed profile by MILP algorithm are $5616.77 J/kg$ and $5025.95 J/kg$, respectively. The simulation result of MILP and actual running trajectory are shown in Figure 1. It can be seen that the driving strategy of driver does not have coasting regime, and the cruise regime is directly switched to the braking regime, but the braking regime is adopted when the train is far from the end point. A reasonable explanation for this driving strategy is that the safety of the train has the first priority in the actual situation, and entering the braking regime in advance can avoid accidents. By comparison, the algorithm proposed in this paper can reduce energy consumption by 10.42% compared with actual energy consumption.

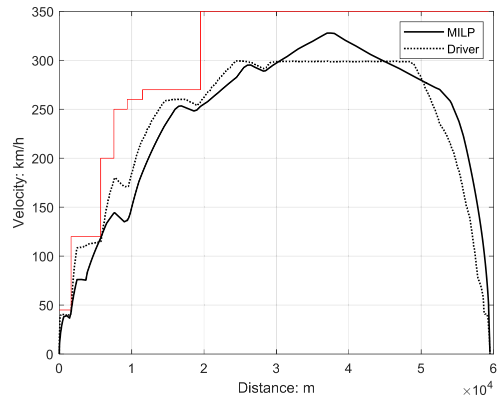


Fig. 1. Compared with the driving curve of the driver.

TABLE IV
STATISTIC RESULT DATA OF CASE 2

Group	Given Time (s)	Neutral Zones	Running Time (s)	Energy (J/kg)
1	1000	without	999.97	5161.07
		with	999.96	5239.06
2	1166	without	1165.91	4014.87
		with	1165.93	4062.24
3	1226	without	1225.98	3633.00
		with	1225.95	3673.57
4	1236	without	1235.96	3573.43
		with	1235.88	3611.34
5	1246	without	1245.99	3512.15
		with	1245.98	3551.76

B. Case 2

First, in order to study the influence of the neutral zones on driving strategy of the train in different given time, five groups of experiments with different given time are designed in this case. Beijing South station to Langfang station is selected as the simulation segment. There are four neutral zones in this segment. The statistic result data is concluded in Table IV. The train trajectories of the group 1 are presented in Figure 2, in which the black dashed line indicates the trajectory without considering neutral zones, and the solid black line represented the trajectory with neutral zones. The control strategies of the group 1 are presented in Figure 3, and a comparison among the five groups is made.

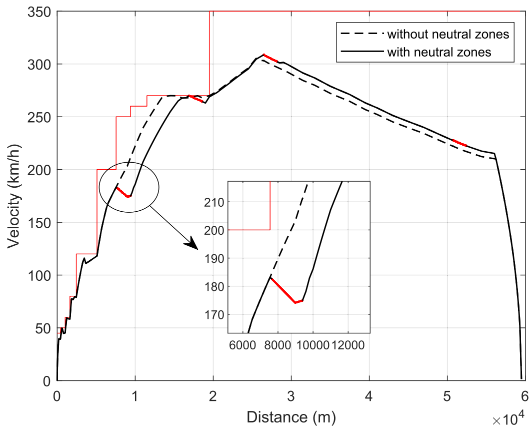
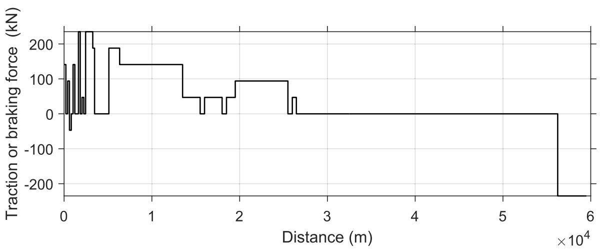
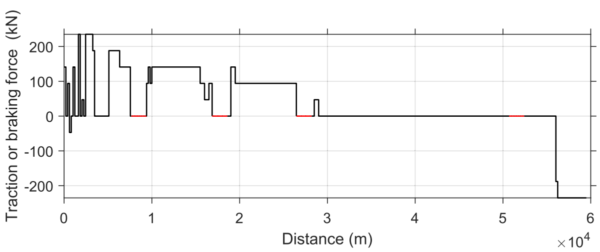


Fig. 2. Trajectories without and with including neutral zones.



(a) Control strategies without considering the neutral zones



(b) Control strategies with considering the neutral zones

Fig. 3. Control strategies without and with considering neutral zones.

Due to the existence of the neutral zones, the traction energy consumption of the train in the 5 groups of experiments has increased by approximately 1.2%. For group 1, the speed of neutral zone is represented by the red line in Figure 2. Specifically, the traction and braking force is zero in the neutral zone, which is marked in red in Figure 3 (b). In addition, it is shown in the Figure 3 (b) that when the train enters the neutral zone, the traction suddenly disappears. This driving strategy aggravates the jerk of the train operation, and the riding comfort will be terrible. This phenomenon can be clearly seen in the first three neutral zones. In particular, when the train runs to the first neutral zone, the traction acceleration of the train suddenly decreases from $0.4 m/s^2$ m to $0 m/s^2$ in a short time. When the train leaves the first neutral zone, the traction acceleration suddenly increases from $0 m/s^2$ to $0.3 m/s^2$.

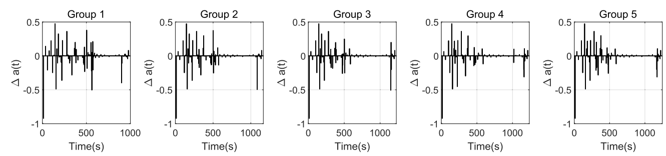
C. Case 3

To deal with the jerk, in this case we analyze the impact of riding comfort on driving strategy when the neutral zones exists. Five experiments which takes the riding comfort into consideration are designed. The route information is the same with that in Case 1. In general, the value of the train acceleration change $\Delta a(t)$ in equation (2) is used as the evaluation index of riding comfort. The results of five groups experiments are presented in Figure 4. And the statistic result data is concluded in Table V.

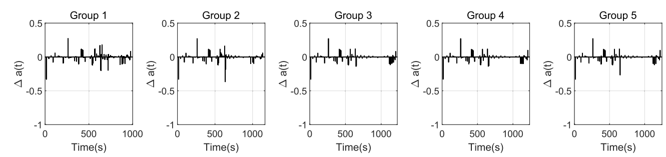
TABLE V

STATISTIC RESULT DATA OF CASE 3

Group	Given Time (s)	Comfort	Maximum $\|\Delta a(t)\|$ (m/s^2)	Energy (J/kg)
1	1000	without	0.92	5239.06
		with	0.33	5879.02
2	1166	without	0.92	4062.24
		with	0.36	4678.81
3	1226	without	0.92	3673.57
		with	0.33	4032.61
4	1236	without	0.92	3611.34
		with	0.33	4047.91
5	1246	without	0.92	3551.76
		with	0.33	3962.20



(a) Without considering the riding comfort

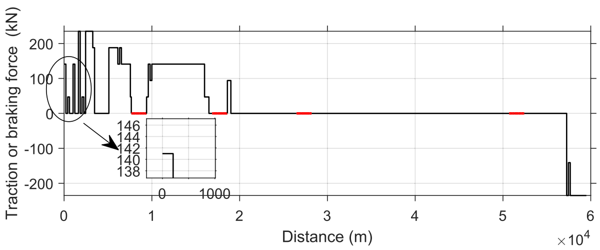


(b) With considering the riding comfort

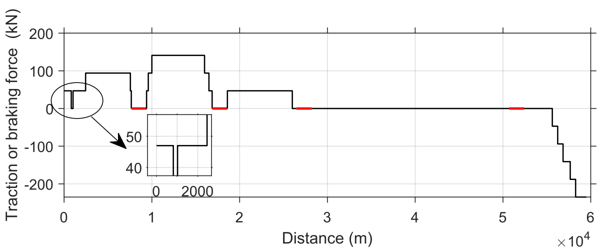
Fig. 4. Simulation result of Case 3.

For the five groups experiments, when the riding comfort is considered, the applied forces of trains only change between successive throttles. By comparing the changes of $\Delta a(t)$ in the comparative experiments, it is clearly observed that the acceleration change range is significantly reduced with considering riding comfort. From the perspective of the control strategy in Figure 5 (a) (such as the group 3), the traction force will change from a large value to zero directly or conversely from zero to a large value when the riding comfort is not considered.

In contrary, the control strategy with considering the riding comfort presents to be gradually decrease until the throttle returns to coasting when the train is approaching the neutral zone. Then, the traction will increase gradually from zero to the maximum available force. This simulation result is also consistent with the driving experience of good drivers who think that smooth switch between throttles can greatly improve the riding comfort. However, the analysis in Table V shows that the energy consumption is increased approximately by 12.16% with considering the riding comfort, which can be explained as the following. The control throttles with considering the riding comfort smoothly switches among the maximum traction and braking force, which goes against with the energy-efficient control strategy (maximum acceleration, cruising, coasting, maximum breaking) in some sense.



(a) Control strategies without considering the riding comfort



(b) Control strategies with considering the riding comfort

Fig. 5. Control strategies without and with considering riding comfort.

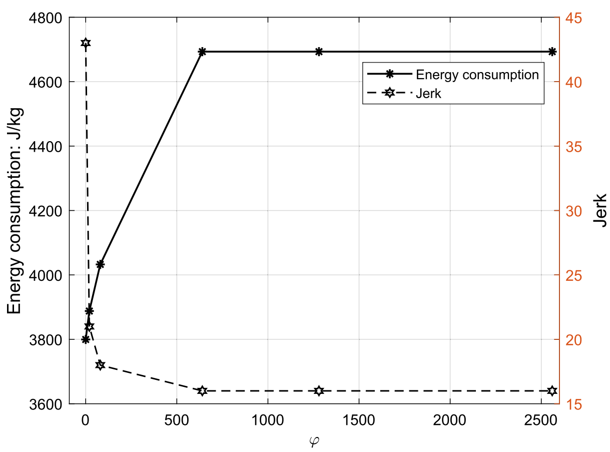


Fig. 6. Relationship among energy consumption, jerk and parameter “φ”.

Specially, the smooth switch of the train throttles means that the average speed in this section is reduced, and the train has to obtain higher speeds in other locations to ensures the given time, so the driving strategy considering the riding comfort will consume more energy.

To explore the sensitiveness of the parameters, the influence of the parameter “φ” on the simulation results is analyzed. The change trend of energy consumption and jerk with parameter “φ” are shown in Figure 6. It is found that the jerk is decreasing with the increasing of the parameter “φ” and the energy consumption is increasing as well. However, it is found that when “φ” is greater than 640, neither energy consumption nor jerk no longer increases or decreases. This result can be explained as the following. The weight of the riding comfort is larger with a bigger value of the parameter “φ”. As a result, the importance of the riding comfort is higher during the optimization process. Specifically, the train will switch less when the riding comfort is considered. When the value of the parameter “φ” is greater than 640, the minimum number of times of driving control throttles switching under the premise of meeting the constraints has been reached. So, the weight of

TABLE VI
RESULT DATA WITHOUT AND WITH CONSIDERING THE TUNNEL

Group	Given Time (s)	φ	Tunnel	Energy (J/kg)
1	1000	200	without	5879.02
			with	7029.02
2	1166	200	without	4678.81
			with	5679.02
3	1200	200	without	4222.89
			with	5232.82
4	1220	200	without	4078.81
			with	4758.40
5	1236	200	without	4047.91
			with	4718.31

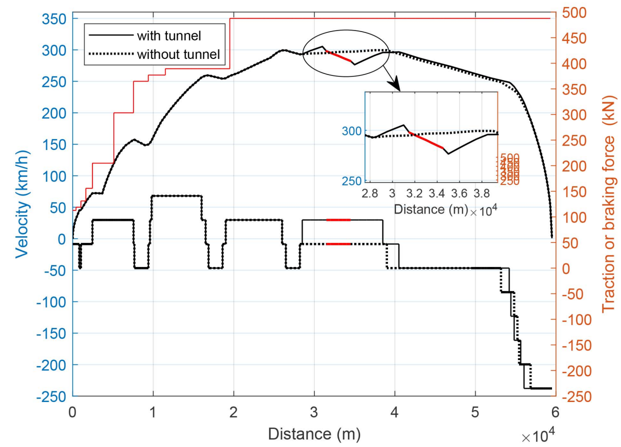


Fig. 7. Simulation result of group 1 in Case 4.

TABLE VII
RESULT WITH DIFFERENT LENGTHS OF THE TUNNEL

Group	Given Time (s)	Position (m)	Length (m)	Energy Consumption (J/kg)
1	1000	31500-34500	3000	7029.02
2	1000	31500-34000	2500	6849.22
3	1000	31500-33500	2000	6779.03
4	1000	31500-33000	1500	6503.51
5	1000	31500-32500	1000	6473.72

the riding comfort during the optimization process is increased, the energy consumption and jerk will not change.

D. Case 4

In this case, we study the influence of the tunnel on the optimal driving strategy of train, but there are no tunnels in the first segment. Then, a virtual tunnel is added in the selected segment from $31500m$ to $34500m$. In equation (20), $a_1', a_2', a_3', c_1', c_2', c_3'$ are set to be $0.012, 0.007, 0.012, 0.00012, 0.00007, 0.00012$, respectively. For comparison, the statistic result data of five groups is concluded in Table VI. It is found that when there is no tunnel on the line, the energy consumption of the train is significantly lower than that when the tunnel is considered in the model. And as the given time increases, the influence of tunnel on energy consumption has a decreasing trend.

TABLE VIII
STATISTIC RESULT OF CASE 5

Group	Interval	φ	Total time (s)	Given time (s)	MILP Running Time (s)	MILP Energy (J/kg)	Total energy (J/kg)
1	Beijing South-Langfang	200	3603	1201	1196.09	4935.96	17024.12
	Langfang-Tianjin South			1201	1200.39	4389.11	
	Tianjin South-Cangzhou West			1201	1191.73	7699.05	
2	Beijing South-Langfang	200	3603	-	1474.25	3129.48	16861.66
	Langfang-Tianjin South			-	871.84	6533.56	
	Tianjin South-Cangzhou West			-	1251.97	7198.62	

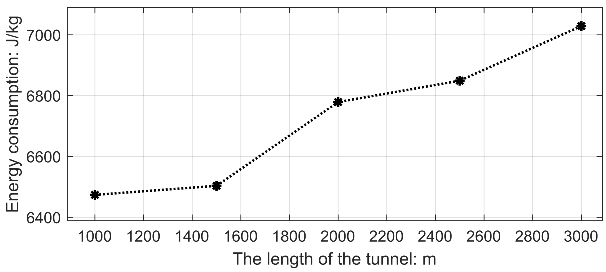


Fig. 8. Energy consumption with different lengths of the tunnel.

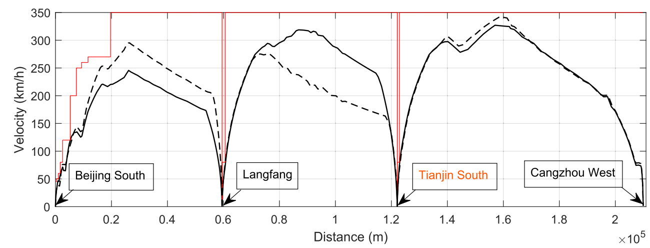


Fig. 9. Trajectories with multiple segments and single segment.

The influence of the tunnel is analyzed as the following. Taking the group 1 of comparative experiments as an example, the simulation result are presented in Figure 7. First, the red solid line in the trajectory in Figure 7 shows that the train speed slows down quickly when the tunnel resistance is considered. Specifically, the train speed is reduced by about $15km/h$ when the train is passing through a 3 kilometers long tunnel. In additional, when the train enters and exits the tunnel, the speed drops more obviously. Second, when the train passes through the tunnel, greater traction force is implemented to compensation the influence of tunnel resistance. This driving strategy consumes more energy. The result can be explained as the following. In order to keep the cruising speed of the train in the tunnel area, the train must output greater traction, which increases the energy consumption of operation. It can be seen from Figure 7 that the train outputs greater traction in the tunnel area, the speed of the train will still decrease. In this case, the energy consumption increased by about 19%.

Further, the influence of the tunnel length on the driving strategy is studied. Then, the train driving strategy experiments with 5 different lengths of the tunnel are designed. The statistic result data is concluded in Table VII. It can be found from Figure 8 that energy consumption increases as the length of the tunnel increases. In addition, there is not a simple linear relationship between energy consumption and tunnel length.

E. Case 5

The method proposed in this paper can be used not only for single segment optimization of train driving strategy, but also for multiple segments optimization. With considering the riding comfort and neutral zones, the driving strategy of train is optimized from the perspective of multiple segments in this case. The total time from Beijing South to Cangzhou West station remains to be $3603s$, but the trip time for each segment is optimized in the case. Then, multiple segments optimization experiments are designed. The statistic result data are concluded in Table VIII. The simulation result of multiple segments optimization (group 2) are presented by black solid line in Figure 9. For the convenience of comparison, the simulation result of single segment optimization (group 1) are also presented by black dotted line in Figure 9.

From the perspective of running time, the total time of the two experiments is the same, but the running time of the first and third segments increased slightly and the second segment is significantly reduced after optimization. On the other hand, although the energy consumption of the second segment has increased by 48%, the other two segments has been reduced. Overall, the total energy consumption has been reduced by 1.1% after the optimization of multiple segments.

TABLE IX
COEFFICIENTS OF THE PWA APPROXIMATION

Method	n	α_n	β_n	E (J)
1	4	$-6.2032 * 10^{-4}$	0.0878	0-100
	5	$-2.2666 * 10^{-5}$	0.0289	100-880
	6	$-1.9047 * 10^{-6}$	0.0122	880-3700
2	4	$-1.1765 * 10^{-3}$	0.1236	0-69
	5	$-9.1050 * 10^{-4}$	0.0489	69-321
	6	$-4.1217 * 10^{-6}$	0.0211	321-3700

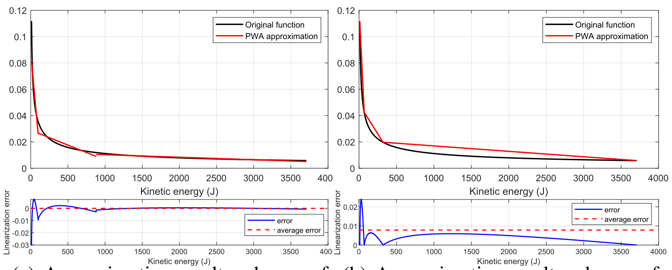
F. Case 6

In this case, the effects of the results of two different linearization methods on driving strategies are analyzed. The parameters of two different approximation methods are concluded in Table IX.

Taking the function $f_1(E(k)) = \frac{1}{2\sqrt{2E}}$ for example, the linearized results and errors of the two different methods are shown in Figure 10. From the perspective of control strategies, the two linearization methods lead to a slight difference between the starting and end positions of the control strategies of trains, and the control strategies of other positions are approximately the same. The statistic result data with different

TABLE X
STATISTIC RESULT DATA WITH DIFFERENT LINEARIZATION METHODS

Method	Given Time (s)	Running Time (s)	Tunnel	Neutral zones	φ	Jerk	Average error	Energy (J/kg)
1	1000	998.06	without	with	200	22	$-4.11 * 10^{-17}$	$5.879 * 10^3$
2	1000	998.37	without	with	200	22	$7.83 * 10^{-3}$	$5.929 * 10^3$



(a) Approximation result and error of the first method
(b) Approximation result and error of the second method

Fig. 10. PWA approximation results and errors of the two different methods.

linearization methods are concluded in Table X. It can be found that the average error of the second linearization method is higher than that of the first linearization method. Comparing the two linearization methods, the train energy consumption obtained by the second linearization method is 0.85% higher than the energy consumption obtained by the first linearization method. Therefore, the linearization error will have an impact on the train control strategies.

VI. CONCLUSION AND DISCUSSION

In this paper, a train trajectory optimization problem for high-speed trains is studied to realize a trade-off between energy consumption and the riding comfort. With considering the neutral zones, tunnel resistance, and discrete throttle settings, a nonlinear optimization model is developed. To solve the problem conveniently and efficiently, the problem is reformulated to be a MILP problem by properties and PWA approximation.

Some numerical cases of Beijing-Shanghai high-speed line are conducted based on the real-world operation data to illustrate the effectiveness of the proposed approach. First, the optimization results of the algorithm proposed in this paper are 10.42% less than the actual train energy consumption. Second, this paper analyzes the impact of neutral zones in the high-speed line on driving strategy. It is found that the train increases energy consumption by approximately 1.2% with the neutral zones are considered in the model. Third, if only improving the riding comfort of passengers, the energy consumption of the train will increase by 2.31% to 23.51%. Fourth, the additional virtual tunnel in the Case 3 brings an extra 19% energy consumption to the train driving strategy. Furthermore, a nonlinear relationship between energy consumption and tunnel length is found. Fifth, multiple segments optimization is designed, which can save about 1.1% energy consumption than single segment optimization. Finally, the effects of the results of two different linearization methods on driving strategies are analyzed. The results show that the linearization error will not have much impact on the energy consumption of the train.

The point mass model is used in this paper and the coordinated control of multiple power units of the train will be considered in the future work. In addition, although MILP algorithm performs well in solving the proposed problem, it still has some limitations. The computation time is still very long when the line distance is long. More efficient algorithm will be developed to cope with state space explosion problem.

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