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Adaptive Multigradient Recursive Reinforcement Learning Event-Triggered Tracking Control for Multiagent Systems

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***Abstract*— This article proposes a fault-tolerant adaptive multigradient recursive reinforcement learning (RL) event-triggered tracking control scheme for strict-feedback discrete-time multiagent systems. The multigradient recursive RL algorithm is used to avoid the local optimal problem that may exist in the gradient descent scheme. Different from the existing event-triggered control results, a new lemma about the relative threshold event-triggered control strategy is proposed to handle the compensation error, which can improve the utilization of communication resources and weaken the negative impact on tracking accuracy and closed-loop system stability. To overcome the difficulty caused by sensor fault, a distributed control method is introduced by adopting the adaptive compensation technique, which can effectively decrease the number of online estimation parameters. Furthermore, by using the multigradient recursive RL algorithm with less learning parameters, the online estimation time can be effectively reduced. The stability of closed-loop multiagent systems is proved by using the Lyapunov stability theorem, and it is verified that all signals are semiglobally uniformly ultimately bounded. Finally, two simulation examples are given to show the availability of the presented control scheme.**

***Index Terms*— Event-triggered control, fault-tolerant control, multiagent systems, multigradient recursive reinforcement learning (RL) algorithm.**

I. Introduction

RECENTLY, the cooperative control problem of multiagent systems (MASs) has received considerable attention due to its widespread applications [1]–[7]. For example, an adaptive distributed fault-tolerant control approach with output regulation was proposed for linear MASs in [8]. Liu *et al.* [9] proposed a new distributed cooperative compound tracking scheme for the vehicular platoon using neural networks (NNs). The finite-time fault-tolerant distributed adaptive tracking control problem was solved for high-order nonlinear MASs in [10]. Wu *et al.* [11] proposed a cooperative adaptive control scheme by using the backstepping technique for high-order stochastic nonlinear MASs subject to dead zone. It is noted that the results in [8]–[11] mainly solved the weight vector problem. However, the ideal weight vector problem has not been considered in the above achievements. Therefore, it is important to design an adaptive control algorithm to search for the optical solution.

The gradient descent method, which can search the optimal solution, has received considerable attention [12]–[29]. For instance, Bai *et al.* [30] proposed an NN reinforcement learning (RL) control scheme for discrete-time nonstrict-feedback systems. A distributed fault-tolerant adaptive control scheme was put forward for MASs via gradient descent method in [31]. Yang and Jagannathan [32] proposed an adaptive critic control strategy for discrete-time systems via the RL algorithm. However, the methods in [12]–[32] searched the optimal solution from a single point, which is easy to cause the local optimal problem. To resolve it, Bai *et al.* [33] designed a novel multigradient recursive RL algorithm. For a family of nonlinear systems with input saturation, an adaptive multigradient recursive RL NN tracking control scheme was presented in [34], whereas the studied control methods in [33] and [34] need to adjust a large number of parameters online so that much estimation time would be wasted. Thus, we need to design an adaptive multigradient recursive RL algorithm that requires less online estimation time.

On the other hand, the inevitable limitation of network resources is considered to be the main cause of unexpected network-induced phenomenon. Therefore, the event-triggered mechanism as one of the effective ways to reduce the communication burden has attracted much interest [35]–[38]. For instance, Mu *et al.* [39] presented an event-triggered adaptive robust critic control strategy for a kind of nonlinear continuous-time systems. An event-triggered decentralized fuzzy adaptive control scheme was proposed for nonlinear large-scale systems in [40]. For a class of stochastic nonlinear MASs with unmeasurable states, Liang *et al.* [41] designed

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an event-triggered fuzzy bipartite tracking control scheme, and the difficulty caused by unmeasurable states was solved by using the distributed reduced-order observer. The results in [39]–[41] considered the tradeoff between resource saving and control performance. It is known that an unnecessarily large event-triggered threshold will reduce the tracking accuracy and endanger the stability of the closed-loop systems. Therefore, Wang *et al.* [42] found an effective compensation method to weaken the impact of event-triggered threshold on the closed-loop systems' performance. Unfortunately, the measurement error in [42] needs to be bounded by a given value regardless of the size of control signal, which may degrade the system accuracy. Thus, it is meaningful to design a new compensation mechanism under the relative threshold event-triggered control.

The above analysis motivates us to present a fault-tolerant adaptive multigradient recursive RL event-triggered tracking control strategy for nonlinear strict-feedback MASs. The corresponding challenging problems at this work are listed as follows.

1) Different from the gradient descent method [30]–[32], the multigradient recursive RL algorithm is presented in this article, which can avoid the local optimal issue. In addition, the proposed approach with less learning parameters (LLPs) can decrease online estimation time, which is hard to be achieved in [33] and [34].
2) Under the framework of relative threshold event-triggered strategy, a lemma (see Lemma 1) is proposed, which can effectively handle the compensation error caused by the operator.
3) A new relative threshold compensation strategy is proposed for event-triggered mechanism, which can improve the tracking accuracy and system stability while reducing the communication burden.

The rest of this article is organized as follows. The priori knowledge is given in Section II. An event-triggered-based multigradient recursive RL control strategy is presented in Section III. Section IV shows some results with respect to practical and numerical examples. Finally, conclusions are given in Section V.

Notation: $\mathbb{R}^m$ $(m = 1, 2, \ldots, n)$ represents the m-dimensional Euclidean space. $|\bullet|$ and $\|\bullet\|$ stand for the absolute value and the Euclidean norm of vector or matrix, respectively. ∇ and grad denote the gradient at some point. $\triangle$ is the differentiate operation.

II. Preliminaries

A. Graph Theory

The information exchange among agents is usually represented by a digraph. The digraph is expressed as $\mathcal{G} = (\mathcal{T}, \mathcal{H}, \mathcal{A})$ with $\mathcal{T} = (1, \ldots, N)$ denoting the nonempty node set, $\mathcal{H} \subseteq \mathcal{T} \times \mathcal{T}$ representing the edge set, and $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{N\times N}$ being the relevant adjacency matrix. Edge $(\mathcal{T}_j, \mathcal{T}_i) \in \mathcal{H}$ denotes an edge from i to j. In addition, if the information flows from node j to i, $a_{i,j} > 0$; otherwise, $a_{i,j} = 0$. $\Theta = \text{diag}\{\theta_1, \ldots, \theta_N\}$ stands for the diagonal matrix with $\theta_i = \sum_{j=1}^{N} a_{i,j}$. $\mathcal{C} = \Theta - \mathcal{A}$ is the Laplacian matrix. $\mathcal{V}_i = \{\mathcal{T}_j | (\mathcal{T}_j, \mathcal{T}_i) \in \mathcal{H}, i \neq j\}$ stands for the neighbors set of the agents.

The augmented graph is expressed as $\bar{\mathcal{G}} = (\bar{\mathcal{T}}, \bar{\mathcal{H}})$ with $\bar{\mathcal{T}} = (\mathbf{0}, 1, \ldots, N)$, $\bar{\mathcal{H}} = \bar{\mathcal{T}} \times \bar{\mathcal{T}}$, where $\mathbf{0}$ is the signal of leader. $\varsigma_{i,\mathbf{0}} > 0$ means that the leader $\mathbf{0}$ can send information to node i; otherwise, $\varsigma_{i,\mathbf{0}} = 0$.

Assumption 1 [41]: For the leader–follower MASs, the digraph $\mathcal{G}$ contains at least a spanning tree with node $\mathbf{0}$ as the root.

B. Problem Formulation

This article considers the discrete-time MASs as

$$\begin{cases} s_{i,1}(k+1) = \tau_{i,1}s_{i,2}(k) + g_{i,1}(\bar{s}_{i,1}(k)) \\ s_{i,m}(k+1) = \tau_{i,m}s_{i,m+1}(k) + g_{i,m}(\bar{s}_{i,m}(k)) \\ s_{i,n}(k+1) = \tau_{i,n}\nu_i(k) + g_{i,n}(\bar{s}_{i,n}(k)) + d_i(k) \\ y_i(k) = F(s_{i,1}(k)) \end{cases} \tag{1}$$

where $\bar{s}_{i,m}(k) = [s_{i,1}(k), s_{i,2}(k), \ldots, s_{i,m}(k)]^T \in \mathbb{R}^m$ $(2 \leq m \leq n-1)$ and $\bar{s}_{i,n}(k) = [s_{i,1}(k), s_{i,2}(k), \ldots, s_{i,n}(k)]^T \in \mathbb{R}^n$ represent the state vectors. $g_{i,l}(\bar{s}_{i,l}(k))$ stand for the unknown smooth nonlinear functions, $\tau_{i,l} > 0$ denote the known parameters, and $d_i(k)$ represent the unknown external disturbances with $l = 1, \ldots, n$ and $i = 1, \ldots, N$. $\nu_i(k) \in R$ and $y_i(k) \in R$ denote the input and output of the MASs, respectively. We define $y_d(k)$ as the leader's dynamic. In addition, $F(s_{i,1}(k)) = \check{\zeta}_i(k)s_{i,1}(k) + \imath_i(k)$ stands for the sensor fault with $\check{\zeta}_i(k)$ and $\imath_i(k)$ being the parameters of sensor fault.

Based on the analysis of sensor fault in [43] and [44], the sensor fault model is introduced in discrete-time MASs. The parameters of the sensor fault considered in this article satisfy $\underline{\check{\zeta}} \leq \check{\zeta}_i(k) \leq 1$ and $\underline{\imath} \leq \imath_i(k) \leq \bar{\imath}$ with $\underline{\check{\zeta}} > 0$, $\underline{\imath}$ and $\bar{\imath}$ being the lower and upper bounds of $\imath_i(k)$, respectively. According to the above analysis, the following four types of faults are considered in this article.

1) If $\check{\zeta}_i(k) = 1$ and $\imath_i(k)$ is a constant, the fault is called the bias fault.
2) If $\check{\zeta}_i(k) = 1$ and $|\imath_i(k)| = \varsigma k$ with $0 < \varsigma \ll 1$, the fault is called the drift fault.
3) If $\check{\zeta}_i(k) = 1$ and $|\imath_i(k)| < \bar{\imath}$ with $\triangle\imath_i(k) \to 0$, the fault is named loss of accuracy.
4) If $\underline{\check{\zeta}} \leq \check{\zeta}_i(k) < 1$ and $\imath_i(k) = 0$, the fault is called loss of effectiveness.

Thus, the sensor fault is organized as

$$y_i(k) = s_{i,1}(k) + f_i(k) \tag{2}$$

with $f_i(k) = (\check{\zeta}_i(k) - 1)s_{i,1}(k) + \imath_i(k)$.

On the other hand, inspired by [42], a new relative threshold strategy-based event-triggered scheme is introduced. Fig. 1 shows the block diagram of control scheme, and we define the transmission error as

$$\check{\upsilon}_i(k) = u_i^t(k) - u_i(k) = u_i(k_b) - u_i(k), \quad k_b \leq k < k_{b+1} \tag{3}$$

where $u_i(k)$ denotes the current feedback control signal and $u_i^t(k) = u_i(k_b)$ represents the last transmitted control signal

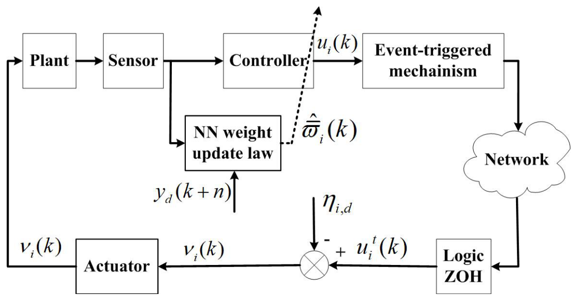

Fig. 1. Block diagram of the event-triggered adaptive control scheme.

with k_b standing for the triggering instant and $b \in N$. Then, the relative threshold control condition is designed as

$$|\breve{\upsilon}_i(k)| \geq \check{\eta}_i(|u_i(k)| + \eta_i) \tag{4}$$

where $0 < \check{\eta}_i < 1$ and $\eta_i > 0$ are designed parameters.

Based on (3) and (4), it yields

$$k_{b+1} = \min\{k \in N | k > k_b, \quad |\breve{\upsilon}_i(k)| \geq \check{\eta}_i(|u_i(k)| + \eta_i)\}.$$

In addition, to degrade the influence of event-triggered threshold on the performance of closed-loop MASs, a designed operator is introduced into the actuator as follows:

$$\nu_i(k) = u_i^t(k) - \eta_{i,d}$$

where $\eta_{i,d}$ will be designed later.

Assumption 2 [31], [45]: The leader $y_d(k)$ and its differences are assumed as bounded functions.

Assumption 3 [46]: The external disturbance $|d_i(k)| \leq \bar{d}$ is assumed to be bounded, and $\bar{d} > 0$ is a constant.

Lemma 1: For a positive constant $\breve{\sigma}_i$ and a time-varying parameter $|\varphi_i(k)| \leq 1$, one has

$$|\varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}| < \check{\eta}_i\eta_i, \quad |\varphi_i(k)| < 1$$
$$|\varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}| \leq 0.2785\breve{\sigma}_i, \quad |\varphi_i(k)| = 1$$

where $\eta_{i,d}$ is designed in (27).

Proof: Since $|\varphi_i(k)| \leq 1$, we discuss it in five cases as follows.

1) When $0 < \varphi_i(k) < 1$, we can get $\text{sgn}(\varphi_i(k)) = 1$, and thus, it follows that

$$|\varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}| = \check{\eta}_i\eta_i\left|\varphi_i(k) - \tanh\left(\frac{\check{\eta}_i\eta_i}{\breve{\sigma}_i}\right)\right| \leq \check{\eta}_i\eta_i.$$

2) When $\varphi_i(k) = 0$, we can get $\text{sgn}(\varphi_i(k)) = 0$, and thus, it follows that

$$|\varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}| = 0.$$

3) When $-1 < \varphi_i(k) < 0$, we can get $\text{sgn}(\varphi_i(k)) = -1$, and thus, it follows that

$$|\varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}| = \check{\eta}_i\eta_i\left|\varphi_i(k) + \tanh\left(\frac{\check{\eta}_i\eta_i}{\breve{\sigma}_i}\right)\right| \leq \check{\eta}_i\eta_i.$$

4) When $\varphi_i(k) = 1$, we can get $\text{sgn}(\varphi_i(k)) = 1$, and thus, according to [36], it follows that

$$|\varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}| = \left|\check{\eta}_i\eta_i - \check{\eta}_i\eta_i\tanh\left(\frac{\check{\eta}_i\eta_i}{\breve{\sigma}_i}\right)\right| \leq 0.2785\breve{\sigma}_i.$$

5) When $\varphi_i(k) = -1$, we can get $\text{sgn}(\varphi_i(k)) = -1$, and thus, one has

$$|\varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}| = \left|-\check{\eta}_i\eta_i + \check{\eta}_i\eta_i\tanh\left(\frac{\check{\eta}_i\eta_i}{\breve{\sigma}_i}\right)\right| \leq 0.2785\breve{\sigma}_i.$$

Then, based on the above proof, we can get the conclusion of Lemma 1.

According to [47] and (2), one has

$$\begin{cases} s_{i,1}(k+n) = \tau_{i,1}s_{i,2}(k+n-1) + G_{i,1}(\bar{s}_{i,n}(k)) \\ s_{i,m}(\breve{k}-m) = \tau_{i,m}s_{i,m+1}(k+n-m) \\ \qquad\qquad + G_{i,m}(\bar{s}_{i,n}(k)) \\ s_{i,n}(k+1) = \tau_{i,n}\nu_i(k) + G_{i,n}(\bar{s}_{i,n}(k)) + d_i(k) \\ y_i(k) = s_{i,1}(k) + f_i(k) \end{cases} \tag{5}$$

where $\breve{k} = k+n+1$ and $G_{i,1}(\cdot)$, $G_{i,m}(\cdot)$, and $G_{i,n}(\cdot)$ are new functions with $2 \leq m \leq n-1$.

The control goal of this article is to develop an adaptive multigradient recursive RL event-triggered scheme to ensure that all signals of the closed-loop MASs are semiglobally uniformly ultimately bounded (SGUUB).

The distributed errors and the change of coordinates are defined as

$$\begin{cases} \xi_{i,1}(k+n) = \sum_{j\in\mathcal{V}_i} a_{i,j}(y_i(k+n) - y_j(k+n)) \\ \qquad + \varsigma_{i,\mathbf{0}}(y_i(k+n) - y_d(k+n)) \\ \xi_{i,m}(\breve{k}-m) = s_{i,m}(k+n+1-m) - \breve{\varrho}_{i,m-1}(k) \\ \xi_{i,n}(k+1) = s_{i,n}(k+1) - \breve{\varrho}_{i,n-1}(k) \end{cases} \tag{6}$$

with $\varsigma_{i,\mathbf{0}} \geq 0$ standing for the pinning gain and $\breve{\varrho}_{i,m-1}(k)$ and $\breve{\varrho}_{i,n-1}(k)$ being the virtual control signals.

C. Radial Basis Function NNs

The unknown nonlinear function $\varrho(\Lambda)$ is estimated by using the following radial basis function (RBF) NN:

$$\varrho(\Lambda) = \varpi^{*T}\phi(\Lambda)$$

with $\varpi^* \in \mathbb{R}^\beta$ standing for the weight vector and $\phi(\Lambda) = [\phi_1(\Lambda), \phi_2(\Lambda), \ldots, \phi_\beta(\Lambda)]^T$ denoting the basis function vector, where $\beta > 1$ represents the node number of NNs. We choose the Gaussian function form of $\phi_i(\Lambda)$ as

$$\phi_i(\Lambda) = \exp\left[-\frac{(\Lambda - \check{\nu}_i)^T(\Lambda - \check{\nu}_i)}{\check{\tau}_i^2}\right], \quad i = 1, 2, \ldots, \beta$$

where $\check{\nu}_i$ stands for the receptive field center and $\check{\tau}_i$ represents the Gaussian function width.

Obviously, we can get

$$\phi_i^T(\Lambda)\phi_i(\Lambda) \leq \beta.$$

Then, based on [48]–[53], for a constant $\bar{\varepsilon} > 0$ and any unknown function $\varrho(\Lambda)$ over a compact set, one has

$$\varrho(\Lambda) = \varpi^T\phi(\Lambda) + \varepsilon(\Lambda)$$

where ϖ represents the ideal weight vector and $\varepsilon(\Lambda)$ stands for the approximation error with $|\varepsilon(\Lambda)| \leq \bar{\varepsilon}$.

III. Adaptive Multigradient Recursive RL Control Design

A. Designs of Strategic Utility Function and Critic NNs

Based on [31], we define

$$\Upsilon_i(k) = \begin{cases} 0, & |\xi_{i,1}(k)| \le \breve{\upsilon} \\ 1, & |\xi_{i,1}(k)| > \breve{\upsilon} \end{cases}$$

where $\breve{\upsilon}$ denotes the tracking performance threshold and $\Upsilon_i(k)$ represents the tracking performance index.

Then, from [54], the long-term strategic utility function (SUF) can be designed as

$$Q_i(k) = \Upsilon_i(k+1) + \breve{\delta}_i \Upsilon_i(k+2) + \breve{\delta}_i^2 \Upsilon_i(k+3) + \dots$$

where $\breve{\delta}_i > 0$ denotes a designed weighting parameter.

Since $Q_i(k)$ is unknown, critic NNs are used to approximate it, and one has

$$Q_i(k) = \varpi_{i,o}^T \phi_{i,o}(k) + \varepsilon_{i,o}(k)$$

where $\varpi_{i,o}$ stands for the ideal weight vector and $\varepsilon_{i,o}(k)$ represents the approximation error.

Based on the idea of LLPs in [55], it yields

$$\hat{Q}_i(k) = \hat{\bar{\varpi}}_{i,o}(k)\bar{\phi}_{i,o}(k) \tag{7}$$

with $\hat{Q}_i(k)$ being the estimation of $Q_i(k)$, $\hat{\bar{\varpi}}_{i,o}$ being the estimation of $\bar{\varpi}_{i,o} = \|\varpi_{i,o}\|$, and $\bar{\phi}_{i,o}(k) = \|\phi_{i,o}(k)\|$.

Remark 1: To obtain better control performance, the number of NN nodes in the hidden layer may be large, and then, more computation time is required. Therefore, the method with LLPs that adjusts the Euclidean norm of weights is used in this article to reduce the online computation time.

Similar to [56]–[58], we define the prediction error as

$$\xi_{i,o}(k) = \breve{\delta}_i \hat{Q}_i(k) - [\hat{Q}_i(k-1) - \Upsilon_i(k)].$$

Then, we define the cost function (CF) as $\Theta_{i,o}(k) = \xi_{i,o}^2(k)/2$, and the gradient can be derived as

$$\begin{aligned}\nabla\hat{\bar{\varpi}}_{i,o}(k) &= \frac{\partial \Theta_{i,o}(k)}{\partial \hat{\bar{\varpi}}_{i,o}(k)} \\ &= \breve{\delta}_i \bar{\phi}_{i,o}(k)[\breve{\delta}_i \hat{Q}_i(k) - \hat{Q}_i(k-1) + \Upsilon_i(k)].\end{aligned}$$

According to [34], the multigradient can be obtained as

$$\begin{aligned}\mathrm{grad}(\rho, \Theta_{i,o}(k)) = \sum_{l=1}^{\rho} &\breve{\delta}_i \bar{\phi}_{i,o}(k-l+1) \\ &\times [\breve{\delta}_i \chi_{i,o}(k-l+1) - \chi_{i,o}(k-l) \\ &+ \Upsilon_i(k-l+1)]\end{aligned} \tag{8}$$

where ρ denotes the gradient length, $\chi_{i,o}(k-l+1) = \hat{\bar{\varpi}}_{i,o}(k)\bar{\phi}_{i,o}(k-l+1)$, and $\chi_{i,o}(k-l) = \hat{\bar{\varpi}}_{i,o}(k)\bar{\phi}_{i,o}(k-l)$.

The multigradient recursive updating law can be designed as

$$\begin{aligned}\hat{\bar{\varpi}}_{i,o}(k+1) = \hat{\bar{\varpi}}_{i,o}(k) - \zeta_{i,o} \sum_{l=1}^{\rho} &\breve{\delta}_i \bar{\phi}_{i,o}(k-l+1) \\ &\times [\breve{\delta}_i \chi_{i,o}(k-l+1) - \chi_{i,o}(k-l) \\ &+ \Upsilon_i(k-l+1)]\end{aligned} \tag{9}$$

where $\zeta_{i,o} > 0$ represents the convergence factor.

B. Design of Adaptive RL Controller

By using the direct adaptive control method, an event-triggered-based distributed NN multigradient recursive RL controller will be designed in this section.

Step 1: Based on (5) and (6), it yields

$$\begin{aligned}\xi_{i,1}(k+n) = &(\theta_i + \varsigma_{i,\mathbf{0}})(G_{i,1}(\bar{s}_{i,n}(k)) + \tau_{i,1}s_{i,2}(k+n-1) \\ &+ f_i(k+n)) - \sum_{j\in\mathcal{V}_i} a_{i,j} \\ &\times (G_{j,1}(\bar{s}_{j,n}(k)) + \tau_{j,1}s_{j,2}(k+n-1) \\ &+ f_j(k+n)) - \varsigma_{i,\mathbf{0}} y_d(k+n).\end{aligned} \tag{10}$$

Inspired by the idea of direct adaptive control method in [59], the desired control signal is designed as

$$\begin{aligned}\varrho_{i,1}(k) = -\frac{1}{\tau_{i,1}(\theta_i + \varsigma_{i,\mathbf{0}})}\Bigg[&(\theta_i + \varsigma_{i,\mathbf{0}})(G_{i,1}(\bar{s}_{i,n}(k) + f_i(k+n)) \\ &- \sum_{j\in\mathcal{V}_i} a_{i,j}(G_{j,1}(\bar{s}_{j,n}(k)) \\ &+ \tau_{j,1}s_{j,2}(k+n-1) + f_j(k+n)) \\ &- \varsigma_{i,\mathbf{0}} y_d(k+n)\Bigg].\end{aligned}$$

Since the desired control signal contains the unknown continuous functions $G_{i,1}(\bar{s}_{i,n}(k))$, $G_{j,1}(\bar{s}_{j,n}(k))$, $f_i(k+n)$, and $f_j(k+n)$, the RBF NNs are employed to approximate it, that is,

$$\varrho_{i,1}(k) = \varpi_{i,1}^T \phi_{i,1}(\omega_{i,1}(k)) + \varepsilon_{i,1}(k) \tag{11}$$

where $\omega_{i,1}(k) = [\bar{s}_{i,n}^T(k), y_d(k+n)]^T$. Then, based on (6), (10), and (11), one can obtain

$$\begin{aligned}\xi_{i,1}(k+n) = (\theta_i + \varsigma_{i,\mathbf{0}})\tau_{i,1}(&\xi_{i,2}(k+n-1) \\ &+ \breve{\varrho}_{i,1}(k) - \varrho_{i,1}(k)).\end{aligned}$$

Remark 2: The existing results use dynamic programming methods to solve nonlinear optimization problems, and however, dynamic programming methods have two disadvantages: OFF-line computation and large computation. Therefore, the RL algorithm is introduced in this article and the NNs are used to approximate nonlinear functions online. This algorithm can make up for the shortcomings of dynamic programming.

By using the approach with LLPs, we design the virtual control signal as

$$\breve{\varrho}_{i,1}(k) = \hat{\bar{\varpi}}_{i,1}(k)\bar{\phi}_{i,1}(\omega_{i,1}(k)) \tag{12}$$

with $\hat{\bar{\varpi}}_{i,1}$ being the estimate of $\bar{\varpi}_{i,1} = \|\varpi_{i,1}\|$ and $\bar{\phi}_{i,1} = [\|\phi_{1,1}\|, \dots, \|\phi_{N,1}\|]^T$.

In the light of (11) and (12), we have

$$\begin{aligned}\xi_{i,1}(k+n) = &(\theta_i + \varsigma_{i,\mathbf{0}})\tau_{i,1}\xi_{i,2}(k+n-1) \\ &+ (\theta_i + \varsigma_{i,\mathbf{0}})\tau_{i,1}\tilde{\bar{\varpi}}_{i,1}(k)\bar{\phi}_{i,1}(\omega_{i,1}(k)) \\ &+ (\theta_i + \varsigma_{i,\mathbf{0}})\tau_{i,1}\epsilon_{i,1}(k)\end{aligned} \tag{13}$$

where $\tilde{\hat{\bar{\varpi}}}_{i,1}(k) = \hat{\bar{\varpi}}_{i,1}(k) - \bar{\varpi}_{i,1}$ and $\epsilon_{i,1}(k) = \bar{\varpi}_{i,1}\bar{\phi}_{i,1}(\omega_{i,1}(k)) - \varpi_{i,1}^T\phi_{i,1}(\omega_{i,1}(k)) - \varepsilon_{i,1}(k)$. Then, (13) can be converted to

$$\begin{aligned}\xi_{i,1}(k+1) = {} & (\theta_i + \varsigma_{i,\mathbf{0}})\tau_{i,1}\xi_{i,2}(k)\\ & + (\theta_i + \varsigma_{i,\mathbf{0}})\tau_{i,1}\tilde{\hat{\bar{\varpi}}}_{i,1}(k_1)\bar{\phi}_{i,1}(\omega_{i,1}(k_1))\\ & + (\theta_i + \varsigma_{i,\mathbf{0}})\tau_{i,1}\epsilon_{i,1}(k_1)\end{aligned} \tag{14}$$

where $k_1 = k - n + 1$. In addition, we define

$$\xi_{i,c1}(k) = \hat{\bar{\varpi}}_{i,1}(k_1)\bar{\phi}_{i,1}(\omega_{i,1}(k_1)) + [\hat{Q}_i(k) - Q_{i,d}(k)]$$

with $Q_{i,d}(k)$ denoting the desired SUF and being usually chosen as "0" from [34]. Thus, we design the CF as

$$\Theta_{i,1}(k) = \frac{\xi_{i,c1}^2(k)}{2}.$$

Then, the gradient is expressed as

$$\begin{aligned}\nabla\hat{\bar{\varpi}}_{i,1}(k_1) &= \frac{\partial\Theta_{i,1}(k)}{\partial\hat{\bar{\varpi}}_{i,1}(k_1)}\\ &= \bar{\phi}_{i,1}(\omega_{i,1}(k_1))[\hat{\bar{\varpi}}_{i,1}(k_1)\bar{\phi}_{i,1}(\omega_{i,1}(k_1)) + \hat{Q}_i(k)].\end{aligned}$$

Similar to (8), one can obtain

$$\begin{aligned}\mathrm{grad}(\rho, \Theta_{i,1}(k)) = {} & \sum_{l=1}^{\rho}\bar{\phi}_{i,1}(\omega_{i,1}(k_1 - l + 1))\\ & \times [\chi_{i,1}(k_1 - l + 1) + \chi_{i,o}(k - l + 1)]\end{aligned} \tag{15}$$

where $\chi_{i,1}(k_1 - l + 1) = \hat{\bar{\varpi}}_{i,1}(k_1)\bar{\phi}_{i,1}(\omega_{i,1}(k_1 - l + 1))$. Then, the multigradient recursive updating law can be designed as

$$\begin{aligned}\hat{\bar{\varpi}}_{i,1}(k+1) = {} & \hat{\bar{\varpi}}_{i,1}(k_1) - \zeta_{i,1}\sum_{l=1}^{\rho}\bar{\phi}_{i,1}(\omega_{i,1}(k_1 - l + 1))\\ & \times [\chi_{i,1}(k_1 - l + 1) + \chi_{i,o}(k - l + 1)]\end{aligned} \tag{16}$$

where $\zeta_{i,1} > 0$ is the convergence factor.

Step m ($2 \le m \le n-1$): In the light of (5) and (6), we have

$$\begin{aligned}\xi_{i,m}(k + n - m + 1) = {} & G_{i,m}(\bar{s}_{i,n}(k)) + \tau_{i,m}s_{i,m+1}\\ & \times (k + n - m) - \check{\varrho}_{i,m-1}(k).\end{aligned}$$

The desired control signal is designed as

$$\varrho_{i,m}(k) = -\frac{1}{\tau_{i,m}}(G_{i,m}(\bar{s}_{i,n}(k)) - \check{\varrho}_{i,m-1}(k)).$$

Similar to (11), one has

$$\varrho_{i,m}(k) = \varpi_{i,m}^T\phi_{i,m}(\omega_{i,m}(k)) + \varepsilon_{i,m}(k) \tag{17}$$

where $\omega_{i,m}(k) = [\bar{s}_{i,n}^T(k), \check{\varrho}_{i,m-1}(k)]^T$. Then, based on (6), one yields

$$\begin{aligned}\xi_{i,m}(k + n - m + 1) = {} & \tau_{i,m}(\xi_{i,m+1}(k + n - m)\\ & + \check{\varrho}_{i,m}(k) - \varrho_{i,m}(k)).\end{aligned} \tag{18}$$

Inspired by the method with LLPs, we design the virtual control signal $\check{\varrho}_{i,m}(k)$ as

$$\check{\varrho}_{i,m}(k) = \hat{\bar{\varpi}}_{i,m}(k)\bar{\phi}_{i,m}(\omega_{i,m}(k)) \tag{19}$$

where $\hat{\bar{\varpi}}_{i,m}$ is the estimation of $\bar{\varpi}_{i,m} = \|\varpi_{i,m}\|$ and $\bar{\phi}_{i,m} = [\|\phi_{1,m}\|, \ldots, \|\phi_{N,m}\|]^T$.

Substituting (17) and (19) into (18), it gives

$$\begin{aligned}\xi_{i,m}(k + n - m + 1) = {} & \tau_{i,m}\xi_{i,m+1}(k + n - m) + \tau_{i,m}\\ & \times \tilde{\hat{\bar{\varpi}}}_{i,m}(k)\bar{\phi}_{i,m}(\omega_{i,m}(k)) + \tau_{i,m}\epsilon_{i,m}(k),\end{aligned}$$

where $\tilde{\hat{\bar{\varpi}}}_{i,m}(k) = \hat{\bar{\varpi}}_{i,m}(k) - \bar{\varpi}_{i,m}$ and $\epsilon_{i,m}(k) = \bar{\varpi}_{i,m}\bar{\phi}_{i,m}(\omega_{i,m}(k)) - \varpi_{i,m}^T\phi_{i,m}(\omega_{i,m}(k)) - \varepsilon_{i,m}(k)$.

Similar to (14), we can obtain

$$\begin{aligned}\xi_{i,m}(k+1) = {} & \tau_{i,m}\xi_{i,m+1}(k) + \tau_{i,m}\\ & \times \tilde{\hat{\bar{\varpi}}}_{i,m}(k_m)\bar{\phi}_{i,m}(\omega_{i,m}(k_m)) + \tau_{i,m}\epsilon_{i,m}(k_m)\end{aligned}$$

where $k_m = k - n + m$. Then, we define

$$\xi_{i,cm}(k) = \hat{\bar{\varpi}}_{i,m}(k_m)\bar{\phi}_{i,m}(\omega_{i,m}(k_m)) + [\hat{Q}_i(k) - Q_{i,d}(k)].$$

Thus, we design the CF as

$$\Theta_{i,m}(k) = \frac{\xi_{i,cm}^2(k)}{2}.$$

The gradient can be derived as

$$\begin{aligned}\nabla\hat{\bar{\varpi}}_{i,m}(k_m) &= \frac{\partial\Theta_{i,m}(k)}{\partial\hat{\bar{\varpi}}_{i,m}(k_m)}\\ &= \bar{\phi}_{i,m}(\omega_{i,m}(k_m))\\ &\quad \times [\hat{\bar{\varpi}}_{i,m}(k_m) \times \bar{\phi}_{i,m}(\omega_{i,m}(k_m)) + \hat{Q}_i(k)].\end{aligned}$$

Similar to (15), it yields

$$\begin{aligned}\mathrm{grad}(\rho, \Theta_{i,m}(k)) = {} & \sum_{l=1}^{\rho}\bar{\phi}_{i,m}(\omega_{i,m}(k_m - l + 1))\\ & \times [\chi_{i,m}(k_m - l + 1) + \chi_{i,o}(k - l + 1)]\end{aligned} \tag{20}$$

where $\chi_{i,m}(k_m - l + 1) = \hat{\bar{\varpi}}_{i,m}(k)\bar{\phi}_{i,m}(\omega_{i,m}(k_m - l + 1))$. Then, the multigradient recursive updating law can be designed as

$$\begin{aligned}\hat{\bar{\varpi}}_{i,m}(k+1) = {} & \hat{\bar{\varpi}}_{i,m}(k_m) - \zeta_{i,m}\sum_{l=1}^{\rho}\bar{\phi}_{i,m}(\omega_{i,m}(k_m - l + 1))\\ & \times [\chi_{i,m}(k_m - l + 1) + \chi_{i,o}(k - l + 1)]\end{aligned} \tag{21}$$

where $\zeta_{i,m} > 0$ is the convergence factor.

Step n: In the light of (5) and (6), we have

$$\begin{aligned}\xi_{i,n}(k+1) = {} & G_{i,n}(\bar{s}_{i,n}(k)) + \tau_{i,n}\nu_{i,n}(k) + d_i(k)\\ & - \check{\varrho}_{i,n-1}(k).\end{aligned} \tag{22}$$

For the convenience of controller design, the controller $u_i(k)$ is used to express the input signal $\nu_i(k)$ of MASs. According to the event-triggered condition in (4), there is a time-varying parameter $\varphi_i(k)$ that satisfies $|\varphi_i(k)| \le 1$, which makes the following equation holds:

$$u_i^t(k) = (1 + \varphi_i(k)\check{\eta}_i)u_i(k) + \varphi_i(k)\check{\eta}_i\eta_i.$$

Then, $\nu_i(k)$ can be rewritten as

$$\nu_i(k) = (1 + \varphi_i(k)\check{\eta}_i)u_i(k) + \varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d}. \tag{23}$$

Substituting (23) into (22), it gives

$$\begin{aligned}\xi_{i,n}(k+1) = {} & G_{i,n}(\bar{s}_{i,n}(k)) + \tau_{i,n}((1 + \varphi_i(k)\check{\eta}_i)\\ & \times u_i(k) + \varphi_i(k)\check{\eta}_i\eta_i - \eta_{i,d})\\ & + d_i(k) - \check{\varrho}_{i,n-1}(k).\end{aligned}$$

The desired control signal is designed as

$$\varrho_{i,n}(k) = -\frac{1}{\tau_{i,n}(1+\varphi_i(k)\breve{\eta}_i)}(G_{i,n}(\bar{s}_{i,n}(k)) - \breve{\varrho}_{i,n-1}(k)).$$

Following the procedure in Steps 1 and m, one has

$$\varrho_{i,n}(k) = \varpi_{i,n}^T\phi_{i,n}(\omega_{i,n}(k)) + \varepsilon_{i,n}(k) \tag{24}$$

where $\omega_{i,n}(k) = [\bar{s}_{i,n}^T(k), \breve{\varrho}_{i,n-1}(k)]^T$. Then, we can obtain

$$\begin{aligned}\xi_{i,n}(k+1) &= \tau_{i,n}(1+\varphi_i(k)\breve{\eta}_i)(u_i(k) - \varrho_{i,n}(k)) \\ &\quad + \tau_{i,n}\varphi_i(k)\breve{\eta}_i\eta_i - \tau_{i,n}\eta_{i,d} + d_i(k).\end{aligned} \tag{25}$$

Inspired by the method with LLPs, we design the actual control signal as

$$u_i(k) = \hat{\bar{\varpi}}_{i,n}(k)\bar{\phi}_{i,n}(\omega_{i,n}(k)) \tag{26}$$

with $\hat{\bar{\varpi}}_{i,n}$ being the estimate of $\bar{\varpi}_{i,n} = \|\varpi_{i,n}\|$ and $\bar{\phi}_{i,n} = [\|\phi_{1,n}\|, \ldots, \|\phi_{N,n}\|]^T$. In addition, $\eta_{i,d}$ is designed as

$$\eta_{i,d} = \breve{\eta}_i\eta_i \operatorname{sgn}(\varphi_i(k)) \tanh\left(\frac{\breve{\eta}_i\eta_i}{\breve{\sigma}_i}\right) \tag{27}$$

where $\breve{\sigma}_i > 0$ is defined in Lemma 1 and $\operatorname{sgn}(0) = 0$.

Substituting (24), (26), and (27) into (25), one has

$$\begin{aligned}\xi_{i,n}(k+1) &= \tau_{i,n}(1+\varphi_i(k)\breve{\eta}_i)\tilde{\bar{\varpi}}_{i,n}(k)\bar{\phi}_{i,n}(\omega_{i,n}(k)) \\ &\quad + \tau_{i,n}\pi_i(k) + \tau_{i,n}\epsilon_{i,n} + d_i(k)\end{aligned}$$

where $\tilde{\bar{\varpi}}_{i,n}(k) = \hat{\bar{\varpi}}_{i,n}(k) - \bar{\varpi}_{i,n}$, $\pi_i(k) = \varphi_i(k)\breve{\eta}_i\eta_i - \eta_{i,d}$ and $\epsilon_{i,n}(k) = (1+\varphi_i(k)\breve{\eta}_i)\bar{\varpi}_{i,n}\bar{\phi}_{i,n}(\omega_{i,n}(k)) - (1+\varphi_i(k)\breve{\eta}_i)\varpi_{i,n}^T\phi_{i,n}(\omega_{i,n}(k)) - (1+\varphi_i(k)\breve{\eta}_i)\varepsilon_{i,n}(k)$.

Then, we define

$$\xi_{i,cn}(k) = \hat{\bar{\varpi}}_{i,n}(k)\bar{\phi}_{i,n}(\omega_{i,n}(k)) + [\hat{Q}_i(k) - Q_{i,d}(k)].$$

Thus, we design the CF as

$$\Theta_{i,n}(k) = \frac{\xi_{i,cn}^2(k)}{2}.$$

The gradient can be derived as

$$\begin{aligned}\nabla\hat{\bar{\varpi}}_{i,n}(k) &= \frac{\partial\Theta_{i,n}(k)}{\partial\hat{\bar{\varpi}}_{i,n}(k)} \\ &= \bar{\phi}_{i,n}(\omega_{i,n}(k))[\hat{\bar{\varpi}}_{i,n}(k)\bar{\phi}_{i,n}(\omega_{i,n}(k)) + \hat{Q}_i(k)].\end{aligned}$$

Similar to (20), it yields

$$\begin{aligned}\operatorname{grad}(\rho, \Theta_{i,n}(k)) &= \sum_{l=1}^{\rho}\bar{\phi}_{i,n}(\omega_{i,n}(k-l+1)) \\ &\quad \times [\chi_{i,n}(k-l+1) + \chi_{i,o}(k-l+1)]\end{aligned}$$

where $\chi_{i,n}(k-l+1) = \hat{\bar{\varpi}}_{i,n}(k)\bar{\phi}_{i,n}(\omega_{i,n}(k-l+1))$. Then, the multigradient recursive updating law can be designed as

$$\begin{aligned}\hat{\bar{\varpi}}_{i,n}(k+1) &= \hat{\bar{\varpi}}_{i,n}(k) - \zeta_{i,n}\sum_{l=1}^{\rho}\bar{\phi}_{i,n}(\omega_{i,n}(k-l+1)) \\ &\quad \times [\chi_{i,n}(k-l+1) + \chi_{i,o}(k-l+1)]\end{aligned} \tag{28}$$

where $\zeta_{i,n} > 0$ is the convergence factor.

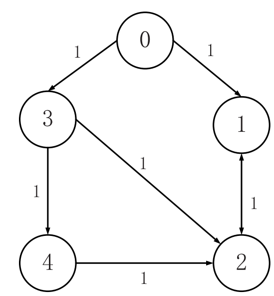

Fig. 2. Communication topology.

C. Stability Analysis

The main theorem of this article is as follows.

Theorem 1: Considering the feasible virtual control signals $\breve{\varrho}_{i,1}(k)$ in (12), $\breve{\varrho}_{i,m}(k)$ in (19), the multigradient recursive updating laws $\hat{\bar{\varpi}}_{i,o}(k+1)$ in (9), $\hat{\bar{\varpi}}_{i,1}(k+1)$ in (16), $\hat{\bar{\varpi}}_{i,m}(k+1)$ in (21), $\hat{\bar{\varpi}}_{i,n}(k+1)$ in (28), and controller $u_i(k)$ in (26), all signals in the closed-loop MASs (1) are SGUUB.

The proof of Theorem 1 can be seen in the Appendix.

IV. Simulation Results

In this section, two simulation examples are addressed to test the effectiveness of the designed algorithm.

We suppose that the nonlinear MASs consist of a leader and four followers. Fig. 2 shows the communication topology with **0** being the leader and 1–4 being four followers.

Example 1: Choose the numerical example as follows:

$$\begin{aligned}s_{i,1}(k+1) &= \frac{0.02s_{i,1}^2(k)}{0.8 + s_{i,1}^2(k)} + \tau_{i,1}s_{i,2}(k) \\ s_{i,2}(k+1) &= \frac{s_{i,2}^2(k)}{0.5 + s_{i,1}^2(k) + s_{i,2}^2(k)} + \tau_{i,2}\nu_i(k) + d_i(k) \\ y_i &= \breve{\zeta}_i(k)s_{i,1}(k) + \iota_i(k)\end{aligned}$$

where

$$d_i(k) = 1.09\cos(s_{i,1}(k))\cos(0.064k)$$

with $i = 1, 2, 3$, and 4. In addition, the desired signal is selected as $y_d(k) = (3/20) + (3/10)\cos(-25\pi/14 + 50kT\pi/299)$.

We define the parameters $s_1(0) = [0.4, 0.2]^T$, $s_2(0) = [0.4, 0.2]^T$, $s_3(0) = [0.4, 0.2]^T$, $s_4(0) = [0.4, 0.2]^T$, $\zeta_{i,o} = 0.0006$, $\zeta_{i,1} = 0.026$, $\zeta_{i,2} = 0.0344$, $\tau_{i,j} = 0.245$, $\breve{\sigma}_i = 0.00035$, $\breve{\zeta}_1(k) = 1$, $\iota_1(k) = 0.3$, $\breve{\zeta}_2(k) = 0.9$, $\iota_2(k) = 0$, $\breve{\zeta}_3(k) = 1$, $\iota_3(k) = -e^{-s_{3,1}(k)}$, $\breve{\zeta}_4(k) = 1$, and $\iota_4(k) = 1$ with $i = 1, 2, 3$, and 4 and $j = 1$ and 2.

The numerical simulation results are given in Figs. 3–6. Fig. 3 shows that the outputs of followers can converge to a neighborhood of the leader's output. The responses of controllers and input signals of MASs are shown in Fig. 4. Fig. 5 shows the event triggering time of u_1–u_4, and it can be seen that the event-triggered controllers can save communication resources. In addition, Fig. 6 shows the trajectories of $\hat{\bar{\varpi}}_{1,o}$, $\hat{\bar{\varpi}}_{1,1}$, and $\hat{\bar{\varpi}}_{1,2}$.

Example 2: To further verify the effectiveness of the proposed control method, a stirred tank reactor practical system

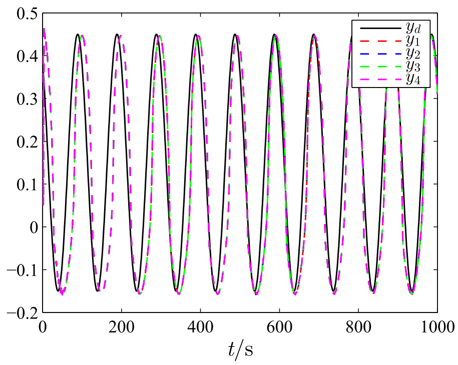

Fig. 3. Trajectories of followers y_i and leader y_d with $i = 1, 2, 3$, and 4 in Example 1.

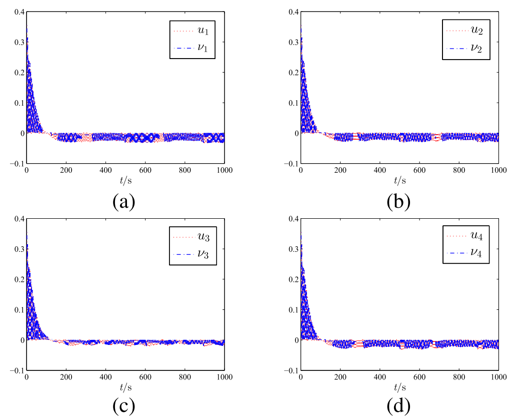

Fig. 4. Controllers and input signals of MASs in Example 1. (a) Trajectories of controller u_1 and input signal ν_1. (b) Trajectories of controller u_2 and input signal ν_2. (c) Trajectories of controller u_3 and input signal ν_3. (d) Trajectories of controller u_4 and input signal ν_4.

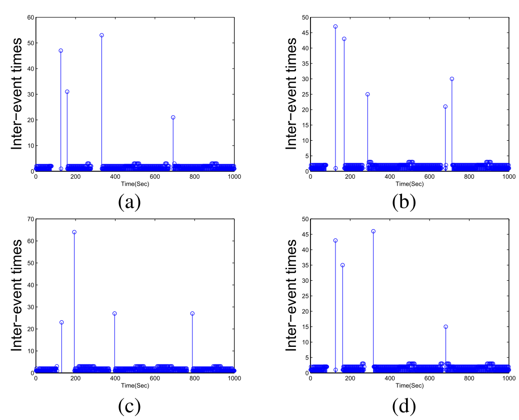

Fig. 5. Triggering time of u_i with $i = 1, 2, 3$, and 4 in Example 1. (a) Triggering time of u_1. (b) Triggering time of u_2. (c) Triggering time of u_3. (d) Triggering time of u_4.

is considered. The same topology is selected as Example 1. In addition, each parameter's physical meaning is shown in Table I, and the system [60] is described as follows:

$$s_{i,1}(k+1) = s_{i,1}(k) + T(-s_{i,1}(k) + \Gamma_{i,1}(k))$$
$$s_{i,2}(k+1) = s_{i,2}(k) + T(-s_{i,2} + \check{\mu}_{i,1}\Gamma_{i,1}(k) - \Gamma_{i,2}(k)) + d_i(k), \quad i = 1, 2, 3, 4$$

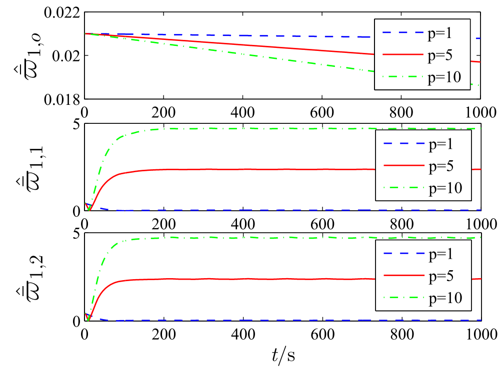

Fig. 6. Trajectories of $\hat{\breve{\varpi}}_{1,o}$, $\hat{\breve{\varpi}}_{1,1}$ and $\hat{\breve{\varpi}}_{1,2}$ in Example 1.

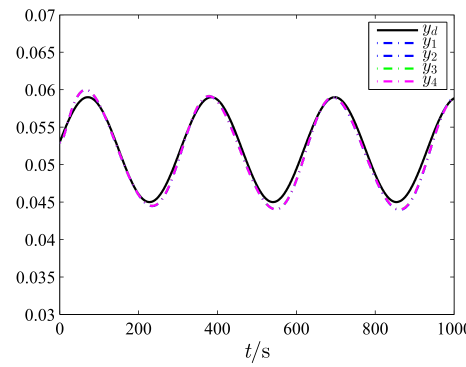

Fig. 7. Trajectories of followers y_i and leader y_d with $i = 1, 2, 3$, and 4 in Example 2.

TABLE I
PARAMETERS OF SIMULATION

$\acute{\sigma}_{i,1} = 0.04$	the Damkohler number
$\acute{\sigma}_{i,2} = 25.2$	the dimensionless cooling rate
$\check{\mu}_{i,1} = 21$	the heat of reaction for dimensionless
$s_{i,1}(k)$	the dimensionless concentration at time k
$s_{i,2}(k)$	the dimensionless temperature
$d_i(k)$	the disturbance
T	the sampling period

with

$$\Gamma_{i,1}(k) = \acute{\sigma}_{i,1}(1.23 - 2.2s_{i,1})e^{\frac{\breve{\varpi}_i s_{i,2}(k)}{\breve{\varpi}_i + s_{i,2}(k)}}$$
$$\Gamma_{i,2}(k) = \acute{\sigma}_{i,2}(s_{i,2}(k) - \nu_i(k))$$

where $\breve{\varpi}_i = 28.5$ and $d_i(k) = (2/25)\cos((1/50)k) \cos(s_{i,1}(k))$. Moreover, the signal of the leader is $y_d(k) = (7/1000)\cos((97/20) + (17/2660)k\pi) + (9/200)$.

We define the parameters $s_1(0) = [0.053, 0.053]^T$, $s_2(0) = [0.053, 0.053]^T$ and $s_3(0) = [0.053, 0.053]^T$, $s_4(0) = [0.053, 0.053]^T$, and the remaining parameters are the same as Example 1.

The practical simulation results are shown in Figs. 7–10. Fig. 7 verifies the tracking control performance of the MASs. Fig. 8 shows the responses of controllers and input signals of MASs. Fig. 9 describes the event triggering time of u_1–u_4. Fig. 10 shows the trajectories of multigradient recursive updating laws. From Figs. 3 to 10, it can be seen that all signals of the discrete-time closed-loop MASs are SGUUB.

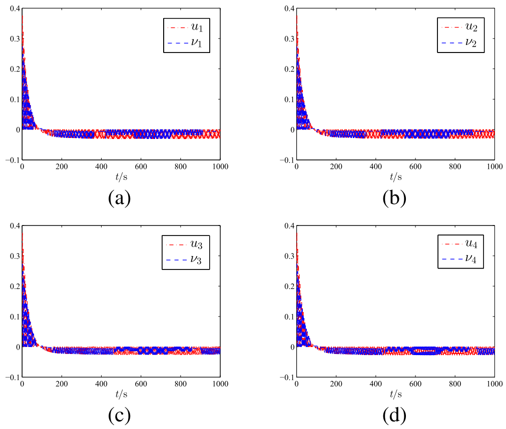

Fig. 8. Controllers and input signals of MASs in Example 2. (a) Trajectories of controller u_1 and input signal ν_1. (b) Trajectories of controller u_2 and input signal ν_2. (c) Trajectories of controller u_3 and input signal ν_3. (d) Trajectories of controller u_4 and input signal ν_4.

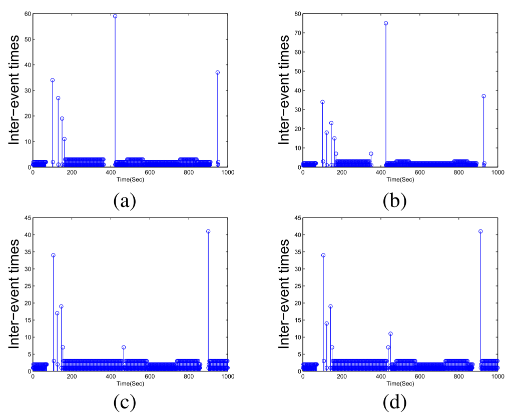

Fig. 9. Triggering time of u_i with $i = 1, 2, 3$, and 4 in Example 2. (a) Triggering time of u_1. (b) Triggering time of u_2. (c) Triggering time of u_3. (d) Triggering time of u_4.

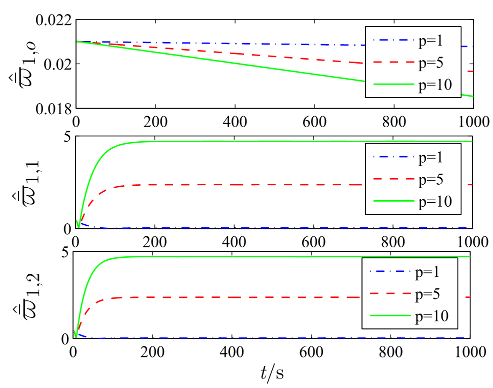

Fig. 10. Trajectories of $\hat{\bar{\varpi}}_{1,o}$, $\hat{\bar{\varpi}}_{1,1}$ and $\hat{\bar{\varpi}}_{1,2}$ in Example 2.

Remark 3: To obtain the desired control performance, select and adjust these parameters under the premise of meeting the design conditions by our experience. In addition, these parameters should be selected according to different system models.

V. Conclusion

In this article, a fault-tolerant adaptive multigradient recursive RL event-triggered consensus control strategy has been presented for strict-feedback discrete-time MASs. The multigradient recursive RL algorithm has been used to avoid the local optimal problem existed in the gradient descent scheme. Then, a new event-triggered relative threshold control strategy with compensation operator has been addressed to improve the utilization of communication resources and weaken the negative impact of the event-triggered threshold on tracking accuracy and stability of closed-loop MASs. A distributed fault-tolerant control method has been used, which can effectively reduce the number of online estimation parameters. Moreover, by combining the direct adaptive control method and the multigradient recursive RL algorithm with LLPs, the computational burden has been effectively reduced. It has been proved that all signals of the closed-loop MASs are SGUUB. At last, the effectiveness of the proposed control scheme has been testified by two simulation examples. Switching threshold event-triggered multigradient recursive RL control problem of discrete-time MASs will be an interesting topic for our further work.

Appendix

This section will give the proof of Theorem 1.

Step 1: The following Lyapunov function candidate is considered:

$$V_{i,1}(k) = V_{i,a1}(k) + V_{i,b1}(k) + V_{i,c1}(k) + V_{i,d1}(k)$$

with

$$\begin{aligned}
V_{i,a1}(k) &= \frac{\alpha_{i,a1}}{3(\theta_i + \varsigma_{i,\mathbf{0}})^2}\xi_{i,1}^2(k),\\
V_{i,b1}(k) &= \frac{\alpha_{i,b1}}{\zeta_{i,1}}\sum_{\check{j}=0}^{n-1}\tilde{\bar{\varpi}}_{i,1}^2(k_1+\check{j}),\\
V_{i,c1}(k) &= \frac{\alpha_{i,o}}{\zeta_{i,o}}\tilde{\bar{\varpi}}_{i,o}^2(k),\\
V_{i,d1}(k) &= 2\alpha_{i,o}\sum_{l=1}^{\rho}e_{i,o}^2(k-l)
\end{aligned}$$

where $\alpha_{i,a1} > 0$, $\alpha_{i,b1} > 0$ and $\alpha_{i,o} > 0$ are constants, and $e_{i,o}(k-l) = \tilde{\bar{\varpi}}_{i,o}(k)\bar{\phi}_{i,o}(k-l)$ with $\tilde{\bar{\varpi}}_{i,o}(k) = \hat{\bar{\varpi}}_{i,o}(k) - \bar{\varpi}_{i,o}$ being the estimation error.

In the light of Cauchy–Schwarz inequality, it yields

$$\begin{aligned}
\triangle V_{i,a1}(k) \le\ & \alpha_{i,a1}\tau_{i,1}^2\xi_{i,2}^2(k) + \alpha_{i,a1}\tau_{i,1}^2 e_{i,1}^2(k_1)\\
&+ \alpha_{i,a1}\tau_{i,1}^2\epsilon_{i,1}^T(k_1)\epsilon_{i,1}(k_1) - \frac{\alpha_{i,a1}}{3(\theta_i+\varsigma_{i,\mathbf{0}})^2}\xi_{i,1}^2(k)\\
\le\ & \alpha_{i,a1}\tau_{i,1}^2\xi_{i,2}^2(k) + \alpha_{i,a1}\tau_{i,1}^2 e_{i,1}^2(k_1)\\
&+ \alpha_{i,a1}\tau_{i,1}^2\bar{\epsilon}_{i,1} - \frac{\alpha_{i,a1}}{3(\theta_i+\varsigma_{i,\mathbf{0}})^2}\xi_{i,1}^2(k),\\
\triangle V_{i,b1}(k) \le\ & -\alpha_{i,b1}(1-\rho\beta\zeta_{i,1})\\
&\times\sum_{l=1}^{\rho}(\chi_{i,1}(k_1-l+1) + \chi_{i,o}(k-l+1))^2\\
&- \alpha_{i,b1}\sum_{l=1}^{\rho}e_{i,1}^2(k_1-l+1)
\end{aligned}$$

$$+ 2\alpha_{i,b1}\sum_{l=1}^{\rho} e_{i,o}^2(k-l+1) + 2\rho\beta\alpha_{i,b1}[\bar{\varpi}_{i,o} + \bar{\varpi}_{i,1}]^2,$$

$$\triangle V_{i,c1}(k) \le -\alpha_{i,o}(1-\rho\beta\zeta_{i,o}\check{o}_i^2) \times \sum_{l=1}^{\rho}(\check{o}_i\chi_{i,o}(k-l+1) + \Upsilon_i(k-l+1) - \chi_{i,o}(k-l))^2 - \alpha_{i,o}\check{o}_i^2\sum_{l=1}^{\rho} e_{i,o}^2(k-l+1) + 2\alpha_{i,o}\sum_{l=1}^{\rho} e_{i,o}^2(k-l) + 2\rho\alpha_{i,o} \times [\bar{\varpi}_{i,o}\beta(1+\check{o}_i)+1]^2$$

$$\triangle V_{i,d1}(k) = 2\alpha_{i,o}\sum_{l=1}^{\rho} e_{i,o}^2(k-l+1) - 2\alpha_{i,o} \times \sum_{l=1}^{\rho} e_{i,o}^2(k-l)$$

where $e_{i,1}(k_1) = \tilde{\bar{\varpi}}_{i,1}(k_1)\bar{\phi}_{i,1}(\omega_{i,1}(k_1))$ and $\epsilon_{i,1}^T(k_1)\epsilon_{i,1}(k_1) \le (\bar{\varepsilon}_{i,1} + 2\sqrt{\beta}\bar{\varpi}_{i,1})^2 = \bar{\epsilon}_{i,1}$ with $\bar{\epsilon}_{i,1} > 0$ being a constant and $\varepsilon_{i,1}(k) \le \bar{\varepsilon}_{i,1}$. Then, one has

$$\begin{aligned}\triangle V_{i,1}(k) &\le -\alpha_{i,b1}(1-\rho\beta\zeta_{i,1}) \times \sum_{l=1}^{\rho}(\chi_{i,1}(k_1-l+1) + \chi_{i,o}(k-l+1))^2 \\ &\quad - \alpha_{i,o}\big(1-\rho\beta\zeta_{i,o}\check{o}_i^2\big) \times \sum_{l=1}^{\rho}(\check{o}_i\chi_{i,o}(k-l+1) + \Upsilon_i(k-l+1) - \chi_{i,o}(k-l))^2 \\ &\quad - \frac{\alpha_{i,a1}}{3(\theta_i+\varsigma_{i,\mathbf{0}})^2}\xi_{i,1}^2(k) - \big(\alpha_{i,b1} - \alpha_{i,a1}\tau_{i,1}^2\big)e_{i,1}^2(k_1) \\ &\quad - \alpha_{i,b1}\sum_{l=2}^{\rho} e_{i,1}^2(k_1-l+1) - \big(\alpha_{i,o}\check{o}_i^2 - 2\alpha_{i,b1} - 2\alpha_{i,o}\big) \\ &\quad \times \sum_{l=1}^{\rho} e_{i,o}^2(k-l+1) + \alpha_{i,a1}\tau_{i,1}^2\xi_{i,2}^2(k) + r_{i,1},\end{aligned} \tag{29}$$

where $r_{i,1} = \alpha_{i,a1}\tau_{i,1}^2\bar{\epsilon}_{i,1} + 2\rho\beta\alpha_{i,b1}[\bar{\varpi}_{i,o} + \bar{\varpi}_{i,1}]^2 + 2\rho\alpha_{i,o}[\bar{\varpi}_{i,o}\beta(1+\check{o}_i)+1]^2$.

Step m ($2 \le m \le n-1$): The following Lyapunov function candidate is considered:

$$V_{i,m}(k) = V_{i,am}(k) + V_{i,bm}(k)$$

with

$$V_{i,am}(k) = \frac{\alpha_{i,am}}{3}\xi_{i,m}^2(k)$$
$$V_{i,bm}(k) = \frac{\alpha_{i,bm}}{\zeta_{i,m}}\sum_{\check{j}=0}^{n-m}\tilde{\bar{\varpi}}_{i,m}^2(k_m+\check{j})$$

where $\alpha_{i,am}$ and $\alpha_{i,bm}$ are positive constants.

In the light of the forward difference, it yields

$$\begin{aligned}\triangle V_{i,am}(k) &\le \alpha_{i,am}\tau_{i,m}^2\xi_{i,m+1}^2(k) + \alpha_{i,am}\tau_{i,m}^2 e_{i,m}^2(k_m) \\ &\quad + \alpha_{i,am}\tau_{i,m}^2\epsilon_{i,m}^T(k_m)\epsilon_{i,m}(k_m) - \frac{\alpha_{i,am}}{3}\xi_{i,m}^2(k) \\ &\le \alpha_{i,am}\tau_{i,m}^2\xi_{i,m+1}^2(k) + \alpha_{i,am}\tau_{i,m}^2 \\ &\quad \times e_{i,m}^2(k_m) + \alpha_{i,am}\tau_{i,m}^2\bar{\epsilon}_{i,m} - \frac{\alpha_{i,am}}{3}\xi_{i,m}^2(k)\end{aligned}$$

$$\begin{aligned}\triangle V_{i,bm}(k) &\le -\alpha_{i,bm}(1-\rho\beta\zeta_{i,m}) \times \sum_{l=1}^{\rho}(\chi_{i,m}(k_m-l+1) + \chi_{i,o}(k-l+1))^2 \\ &\quad - \alpha_{i,bm}\sum_{l=1}^{\rho} e_{i,m}^2(k_m-l+1) + 2\alpha_{i,bm}\sum_{l=1}^{\rho} e_{i,o}^2(k-l+1) \\ &\quad + 2\rho\beta\alpha_{i,bm}[\bar{\varpi}_{i,o} + \bar{\varpi}_{i,m}]^2,\end{aligned}$$

where $e_{i,m}(k_m) = \tilde{\bar{\varpi}}_{i,m}(k_m)\bar{\phi}_{i,m}(\omega_{i,m}(k_m))$ and $\epsilon_{i,m}^T(k_m)\epsilon_{i,m}(k_m) \le (\bar{\varepsilon}_{i,m} + 2\sqrt{\beta}\bar{\varpi}_{i,m})^2 = \bar{\epsilon}_{i,m}$ with $\bar{\epsilon}_{i,m} > 0$ being a constant and $\varepsilon_{i,m} \le \bar{\varepsilon}_{i,m}$. Then, it yields

$$\begin{aligned}\triangle V_{i,m}(k) &\le -\alpha_{i,bm}(1-\rho\beta\zeta_{i,m}) \times \sum_{l=1}^{\rho}(\chi_{i,m}(k_m-l+1) + \chi_{i,o}(k-l+1))^2 \\ &\quad - \frac{\alpha_{i,am}}{3}\xi_{i,m}^2(k) + 2\alpha_{i,bm}\sum_{l=1}^{\rho} e_{i,o}^2(k-l+1) \\ &\quad - \big(\alpha_{i,bm} - \alpha_{i,am}\tau_{i,m}^2\big)e_{i,m}^2(k_m) \\ &\quad - \alpha_{i,bm}\sum_{l=2}^{\rho} e_{i,m}^2(k_m-l+1) \\ &\quad + r_{i,m} + \alpha_{i,am}\tau_{i,m}^2\xi_{i,m+1}^2(k),\end{aligned} \tag{30}$$

where $r_{i,m} = \alpha_{i,am}\tau_{i,m}^2\bar{\epsilon}_{i,m} + 2\rho\beta\alpha_{i,bm}[\bar{\varpi}_{i,o} + \bar{\varpi}_{i,m}]^2$.

Step n: The following Lyapunov function candidate is considered:

$$V_{i,n}(k) = V_{i,an}(k) + V_{i,bn}(k)$$

with

$$V_{i,an}(k) = \frac{\alpha_{i,an}}{4}\xi_{i,n}^2(k)$$
$$V_{i,bn}(k) = \frac{\alpha_{i,bn}}{\zeta_{i,n}}\tilde{\bar{\varpi}}_{i,n}^2(k)$$

where $\alpha_{i,an}$ and $\alpha_{i,bn}$ are positive constants.

On the basis of the forward difference, it yields

$$\begin{aligned}\triangle V_{i,an}(k) &\le \alpha_{i,an}\tau_{i,n}^2(1+\varphi_i(k)\check{\eta}_i)^2 e_{i,n}^2(k) \\ &\quad + \alpha_{i,an}\tau_{i,n}^2\epsilon_{i,n}^T(k)\epsilon_{i,n}(k) + \alpha_{i,an}d_i^2(k) \\ &\quad + \alpha_{i,an}\tau_{i,n}^2\pi_i^2(k) - \frac{\alpha_{i,an}}{4}\xi_{i,n}^2(k) \\ &\le \alpha_{i,an}\tau_{i,n}^2(1+\varphi_i(k)\check{\eta}_i)^2 e_{i,n}^2(k) \\ &\quad + \alpha_{i,an}\tau_{i,n}^2\bar{\epsilon}_{i,n} + \alpha_{i,an}\bar{d}^2 \\ &\quad + \alpha_{i,an}\tau_{i,n}^2\pi_m^2 - \frac{\alpha_{i,an}}{4}\xi_{i,n}^2(k),\end{aligned}$$

$$\triangle V_{i,bn}(k) \le -\alpha_{i,bn}(1-\rho\beta\zeta_{i,n})$$

$$\times\sum_{l=1}^{\rho}(\chi_{i,n}(k-l+1)+\chi_{i,o}(k-l+1))^2$$
$$-\alpha_{i,bn}\sum_{l=1}^{\rho}e_{i,n}^2(k-l+1)$$
$$+2\alpha_{i,bn}\sum_{l=1}^{\rho}e_{i,o}^2(k-l+1)$$
$$+2\rho\beta\alpha_{i,bn}[\bar{\varpi}_{i,o}+\bar{\varpi}_{i,n}]^2$$

where $e_{i,n}(k)=\tilde{\bar{\varpi}}_{i,n}(k)\bar{\phi}_{i,n}(\omega_{i,n}(k))$ and $\epsilon_{i,n}^T(k)\epsilon_{i,n}(k)\le(1+\varphi_i(k)\check{\eta}_i)^2(\bar{\varepsilon}_{i,n}+2\sqrt{\beta}\bar{\varpi}_{i,n})^2=\bar{\epsilon}_{i,n}$ with $\bar{\epsilon}_{i,n}>0$ being a constant and $\varepsilon_{i,n}\le\bar{\varepsilon}_{i,n}$. In addition, by using Lemma 1, it yields that $\pi_i(k)\le\pi_m$ with $\pi_m>0$ being a constant. Then, it gives

$$\begin{aligned}\triangle V_{i,n}(k)\le&-\alpha_{i,bn}(1-\rho\beta\zeta_{i,n})\\&\times\sum_{l=1}^{\rho}(\chi_{i,n}(k-l+1)+\chi_{i,o}(k-l+1))^2\\&-\frac{\alpha_{i,an}}{4}\xi_{i,n}^2(k)+2\alpha_{i,bn}\sum_{l=1}^{\rho}e_{i,o}^2(k-l+1)\\&-\left(\alpha_{i,bn}-\alpha_{i,an}\tau_{i,n}^2(1+\varphi_i(k)\check{\eta}_i)^2\right)e_{i,n}^2(k)\\&-\alpha_{i,bn}\sum_{l=2}^{\rho}e_{i,n}^2(k-l+1)+r_{i,n}\end{aligned}\tag{31}$$

where $r_{i,n}=\alpha_{i,an}\tau_{i,n}^2\bar{\epsilon}_{i,n}+\alpha_{i,an}\bar{d}^2+\alpha_{i,an}\tau_{i,n}^2\pi_m^2+2\rho\beta\alpha_{i,bn}[\bar{\varpi}_{i,o}+\bar{\varpi}_{i,n}]^2$. Then, we let

$$\begin{aligned}V_i(k)=&\sum_{\psi=1}^{n}V_{i,\psi}\\=&\sum_{\psi=1}^{n}\frac{\alpha_{i,b\psi}}{\zeta_{i,\psi}}\sum_{\check{j}=0}^{n-\psi}\tilde{\bar{\varpi}}_{i,\psi}^2(k_\psi+\check{j})+\frac{\alpha_{i,a1}}{3(\theta_i+\varsigma_{i,\mathbf{0}})^2}\\&\times\xi_{i,1}^2(k)+\sum_{\psi=2}^{n-1}\frac{\alpha_{i,a\psi}}{3}\xi_{i,\psi}^2(k)+\frac{\alpha_{i,an}}{4}\xi_{i,n}^2(k)\\&+\frac{\alpha_{i,o}}{\zeta_{i,o}}\tilde{\bar{\varpi}}_{i,o}^2(k)+2\alpha_{i,o}\sum_{l=1}^{\rho}e_{i,o}^2(k-l)\end{aligned}$$

where $k_n=k$.

Consider the Lyapunov function as

$$V(k)=\sum_{i=1}^{N}V_i(k).$$

By combining (29)–(31), we can obtain

$$\begin{aligned}&\triangle V(k)\\&\le-\sum_{i=1}^{N}\sum_{\psi=1}^{n}\alpha_{i,b\psi}(1-\rho\beta\zeta_{i,\psi})\\&\quad\times\sum_{l=1}^{\rho}(\chi_{i,\psi}(k_\psi-l+1)+\chi_{i,o}(k-l+1))^2\\&\quad-\sum_{i=1}^{N}\alpha_{i,o}\left(1-\rho\beta\zeta_{i,o}\check{o}_i^2\right)\\&\quad\times\sum_{l=1}^{\rho}(\check{o}_i\chi_{i,o}(k-l+1)+\Upsilon_i(k-l+1)-\chi_{i,o}(k-l))^2\\&\quad-\sum_{i=1}^{N}\frac{\alpha_{i,a1}}{3(\theta_i+\varsigma_{i,\mathbf{0}})^2}\times\xi_{i,1}^2(k)-\sum_{i=1}^{N}\sum_{\psi=2}^{n-1}\frac{\alpha_{i,a\psi}}{3}\xi_{i,\psi}^2(k)\\&\quad-\sum_{i=1}^{N}\frac{\alpha_{i,an}}{4}\xi_{i,n}^2(k)-\sum_{i=1}^{N}\sum_{\psi=1}^{n-1}\left(\alpha_{i,b\psi}-\alpha_{i,a\psi}\tau_{i,\psi}^2\right)e_{i,\psi}^2(k_\psi)\\&\quad-\sum_{i=1}^{N}\left(\alpha_{i,bn}-\alpha_{i,an}\tau_{i,n}^2(1+\varphi_i(k)\check{\eta}_i)^2\right)e_{i,n}^2(k)\\&\quad-\sum_{i=1}^{N}\sum_{\psi=1}^{n}\alpha_{i,b\psi}\sum_{l=2}^{\rho}e_{i,\psi}^2(k_\psi-l+1)\\&\quad-\sum_{i=1}^{N}\left(\alpha_{i,o}\check{o}_i^2-2\alpha_{i,b1}-2\alpha_{i,o}\right)\sum_{l=1}^{\rho}e_{i,o}^2(k-l+1)\\&\quad+\sum_{i=1}^{N}\sum_{\psi=2}^{n}2\alpha_{i,b\psi}\times\sum_{l=1}^{\rho}e_{i,o}^2(k-l+1)\\&\quad+\sum_{i=1}^{N}\sum_{\psi=1}^{n-1}\alpha_{i,a\psi}\tau_{i,\psi}^2\xi_{i,\psi+1}^2(k)+r\end{aligned}$$

where $r=\sum_{i=1}^{N}r_i$ with $r_i=\sum_{\psi=1}^{n}r_{i,\psi}$.

Choose the proper parameters such that $0<\zeta_{i,o}<(1/(\rho\beta\check{o}_i^2))$, $0<\zeta_{i,\psi}<(1/\rho\beta)$, where $\psi=1,\ldots,n$. Hence, it results in

$$\begin{aligned}\triangle V(k)\le&-\sum_{i=1}^{N}\frac{\alpha_{i,a1}}{3(\theta_i+\varsigma_{i,\mathbf{0}})^2}\xi_{i,1}^2(k)\\&-\sum_{i=1}^{N}\sum_{\psi=2}^{n-1}\left(\frac{\alpha_{i,a\psi}}{3}-\alpha_{i,a(\psi-1)}\tau_{i,(\psi-1)}^2\right)\xi_{i,\psi}^2(k)\\&-\sum_{i=1}^{N}\left(\frac{\alpha_{i,an}}{4}-\alpha_{i,a(n-1)}\tau_{i,(n-1)}^2\right)\xi_{i,n}^2(k)\\&-\sum_{i=1}^{N}\sum_{\psi=1}^{n-1}\left(\alpha_{i,b\psi}-\alpha_{i,a\psi}\tau_{i,\psi}^2\right)e_{i,\psi}^2(k_\psi)\\&-\sum_{i=1}^{N}\left(\alpha_{i,bn}-\alpha_{i,an}\tau_{i,n}^2(1+\varphi_i(k)\check{\eta}_i)^2\right)e_{i,n}^2(k)\\&-\sum_{i=1}^{N}\sum_{\psi=1}^{n}\alpha_{i,b\psi}\sum_{l=2}^{\rho}e_{i,\psi}^2(k_\psi-l+1)\\&-\sum_{i=1}^{N}\left(\alpha_{i,o}\check{o}_i^2-2\alpha_{i,b1}-2\alpha_{i,o}-\sum_{\psi=2}^{n}2\alpha_{i,b\psi}\right)\\&\times\sum_{l=1}^{\rho}e_{i,o}^2(k-l+1)+r.\end{aligned}$$

Clearly, the parameters satisfy

$$\begin{aligned}\alpha_{i,a\psi}&>3\alpha_{i,a(\psi-1)}\tau_{i,(\psi-1)}^2,\quad\psi=2,\ldots,n-1\\\alpha_{i,an}&>4\alpha_{i,a(n-1)}\tau_{i,(n-1)}^2\\\alpha_{i,b\psi}&>\alpha_{i,a\psi}\tau_{i,\psi}^2,\quad\psi=1,\ldots,n-1\end{aligned}$$

$$\alpha_{i,bn} > \alpha_{i,an}\tau_{i,n}^2(1+\varphi_i(k)\check{\eta}_i)^2$$
$$\alpha_{i,o}\check{\delta}_i^2 > 2\alpha_{i,b1} + 2\alpha_{i,o} + \sum_{\psi=2}^{n} 2\alpha_{i,b\psi}.$$

Based on [32], we can get $\triangle V(k) < 0$ from the following conditions:

$$|\xi_{i,1}(k)| > \frac{\sqrt{r}}{\sqrt{\alpha_{i,a1}/(3(\theta_i + \varsigma_{i,\mathbf{0}})^2)}}$$

or

$$|\xi_{i,\psi}(k)| > \frac{\sqrt{r}}{\sqrt{\Phi_1}}, \quad \psi = 2, \ldots, n-1$$

or

$$|\xi_{i,n}(k)| > \frac{\sqrt{r}}{\sqrt{\alpha_{i,an}/4 - \alpha_{i,a(n-1)}\tau_{i,(n-1)}^2}}$$

or

$$|e_{i,\psi}(k)| > \frac{\sqrt{r}}{\sqrt{\alpha_{i,b\psi} - \alpha_{i,a\psi}\tau_{i,\psi}^2}}, \quad \psi = 1, \ldots, n-1$$

or

$$|e_{i,n}(k)| > \frac{\sqrt{r}}{\sqrt{\alpha_{i,bn} - \alpha_{i,an}\tau_{i,n}^2(1+\varphi_i(k)\check{\eta}_i)^2}}$$

or

$$\left|\sum_{l=2}^{\rho} e_{i,\psi}^2(k_\psi - l + 1)\right| > \frac{\sqrt{r}}{\sqrt{\alpha_{i,b\psi}}}, \quad \psi = 1, \ldots, n$$

or

$$\left|\sum_{l=1}^{\rho} e_{i,o}^2(k - l + 1)\right| > \frac{\sqrt{r}}{\sqrt{\Phi_2}}$$

where $\Phi_1 = \alpha_{i,a\psi}/3 - \alpha_{i,a(\psi-1)}\tau_{i,(\psi-1)}^2$, $\Phi_2 = \alpha_{i,o}\check{\delta}_i^2 - 2\alpha_{i,b1} - 2\alpha_{i,o} - \sum_{\psi=2}^{n} 2\alpha_{i,b\psi}$. Then, it means that all signals of closed-loop MASs are SGUUB.

Remark 4: Compared with the results in [30]–[32], the multigradient recursive RL algorithm with LLPs is used in this article, which can avoid the local optimal issue and decrease online estimation time. In addition, different from the traditional relative threshold event-triggered strategy [35]–[41], a compensation operator is introduced in this article, which can reduce the communication burden and weaken the negative impact of the event-triggered threshold on tracking accuracy and stability of closed-loop MASs.

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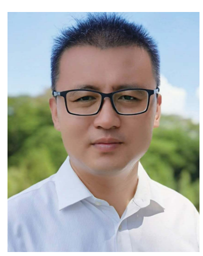

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